14-1100 Oct 16, hour 16: G12, G21, Solvable groups

October-10-14 1:09 PM

Further return HW1

HW2 due!

TT next class, Mon. Oct 20 1:10–3PM here

Material: Everything on groups. See oldies.

Final exam: Mondy Dec 8 Top; Wed Dec 10 Analysis; Thu Dec 11 PDE

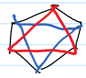
Algebra: Fri Dec 12 or Mon Dec 15? Time?

Groups of order 12. $P_4 = \mathbb{Z}/4$ or $(\mathbb{Z}/2)^2$, $P_3 = \mathbb{Z}/3$, at least one of those is normal, so:

on board

$G = \mathbb{Z}/3 \rtimes \mathbb{Z}/4$: $\text{Aut}(\mathbb{Z}/3) = \mathbb{Z}/2$ so

$\mathbb{Z}/12$ or no-name $\mathbb{Z}/3 \rtimes_{parity} \mathbb{Z}/4$

$G = \mathbb{Z}/3 \rtimes (\mathbb{Z}/2 \times \mathbb{Z}/2)$: $\mathbb{Z}/6 \times \mathbb{Z}/2$ or $S_3 \times \mathbb{Z}/2 = D_{12}$ 

$G = \mathbb{Z}/4 \rtimes \mathbb{Z}/3$ $\text{Aut}(\mathbb{Z}/4) = \mathbb{Z}/2 \Rightarrow \mathbb{Z}/12$

$G = (\mathbb{Z}/2 \times \mathbb{Z}/2) \rtimes \mathbb{Z}/3$ $\text{Aut}(\mathbb{Z}/2 \times \mathbb{Z}/2) = S_3 \Rightarrow$

(direct) $\mathbb{Z}/6 \times \mathbb{Z}/2$

or the fun action of $\mathbb{Z}_3$ on $(\mathbb{Z}/2)^2$, giving A_4

$\langle (234) \rangle$ e, $(12)(34)$, $(13)(24)$, $(14)(23)$

Groups of order 21. $\mathbb{Z}/21$, $\mathbb{Z}/7 \rtimes \mathbb{Z}/3 = \langle x \rangle \rtimes \langle y \rangle$

$\text{Aut}(\mathbb{Z}/7) = \mathbb{Z}/6 = \langle \nu \rangle$; $\nu(x) = x^3$; $y \overset{\phi_0}{\mapsto} \nu^0$ or $\overset{\phi_2}{\nu^2}$ or $\overset{\phi_4}{\nu^4}$

$yxy^{-1} = \underbrace{x}_{\mathbb{Z}/21}$ or $\underbrace{x^2 \text{ or } x^4}_{\text{isomorphic}}$

Exercise: $\phi: H \to \text{Aut}(N)$; $\eta \in \text{Aut}\,H$, $\nu \in \text{Aut}(N)$

$\phi\eta: H \to \text{Aut}(N)$ $\quad (\phi^\nu)_h = \nu^{-1} \circ \phi_h \circ \nu$

$\phi^\nu \in \text{Hom}(H, \text{Aut}(N))$

Then $N \rtimes_\phi H \cong N \rtimes_{\phi\eta} H \cong N \rtimes_{\phi^\nu} H$.

In our case $\phi_4 = \phi_2 \circ \eta$ where $\eta: \mathbb{Z}/3 \to \mathbb{Z}/3$ [illegible]

In our case $\phi_4 = \phi_2 \circ \eta$ where $\eta: \mathbb{Z}/3 \to \mathbb{Z}/3$ is multiplication by 2.

done line

Solvable Groups. <u>Def</u> G is solvable if all quotients in its Jordan-Hölder series are Abelian.

<u>Thm</u> 1. If $N \trianglelefteq G$, G is solvable iff N & G/N are.

2. If $H < G$ and G is solvable, so is H.

$A \trianglelefteq B$ $\quad H \cap A \trianglelefteq H \cap B$? ✓ $\frac{H \cap B}{H \cap A} \longrightarrow B/A$ by $[b]_{H \cap A} \to [b]_A$ is injective.

<u>Cor.</u> If a group contains A_n $n \neq 4$, it is not solvable.

Term test line.

Rings.

Definition 2.1.1. *A **ring** consists of a set R together with binary operations $+$ and $\cdot$ satisfying:*

1. $(R, +)$ *forms an abelian group,*
2. $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ $\forall a, b, c \in R$,
3. $\exists 1 \neq 0 \in R$ *such that* $a \cdot 1 = 1 \cdot a = a$ $\forall a \in R$, *and*
4. $a \cdot (b + c) = a \cdot b + a \cdot c$ *and* $(a + b) \cdot c = a \cdot c + b \cdot c$ $\forall a, b, c \in R$.

Also define: Commutative ring.

Examples. $\mathbb{Z}$, $R[x]$, $M_{n \times n}(R)$

Morphisms, (Examples: 1. $\mathbb{Z} \to \mathbb{Z}/n$ 2. $R \to R[x]$ at deg 0 3. $R \to M_{n \times n}(R)$ as diag 4. $ev_u: R[x] \to R$ (if R is commutative) 5. $M_{n \times n}(R[x]) \cong M_{n \times n}(R)[x]$)

im, subring, ker, ideal.

Q. Is every ideal a quotient.

Ans. Define R/I.

Good luck w/ term test!