14-1100 Dec 1, hours 35-36: Topological insolubility of the quintic, more on the JCF

September-03-14 12:34 PM

Pam needs volunteers to mark olympiad questions!

Course evals: 5/17

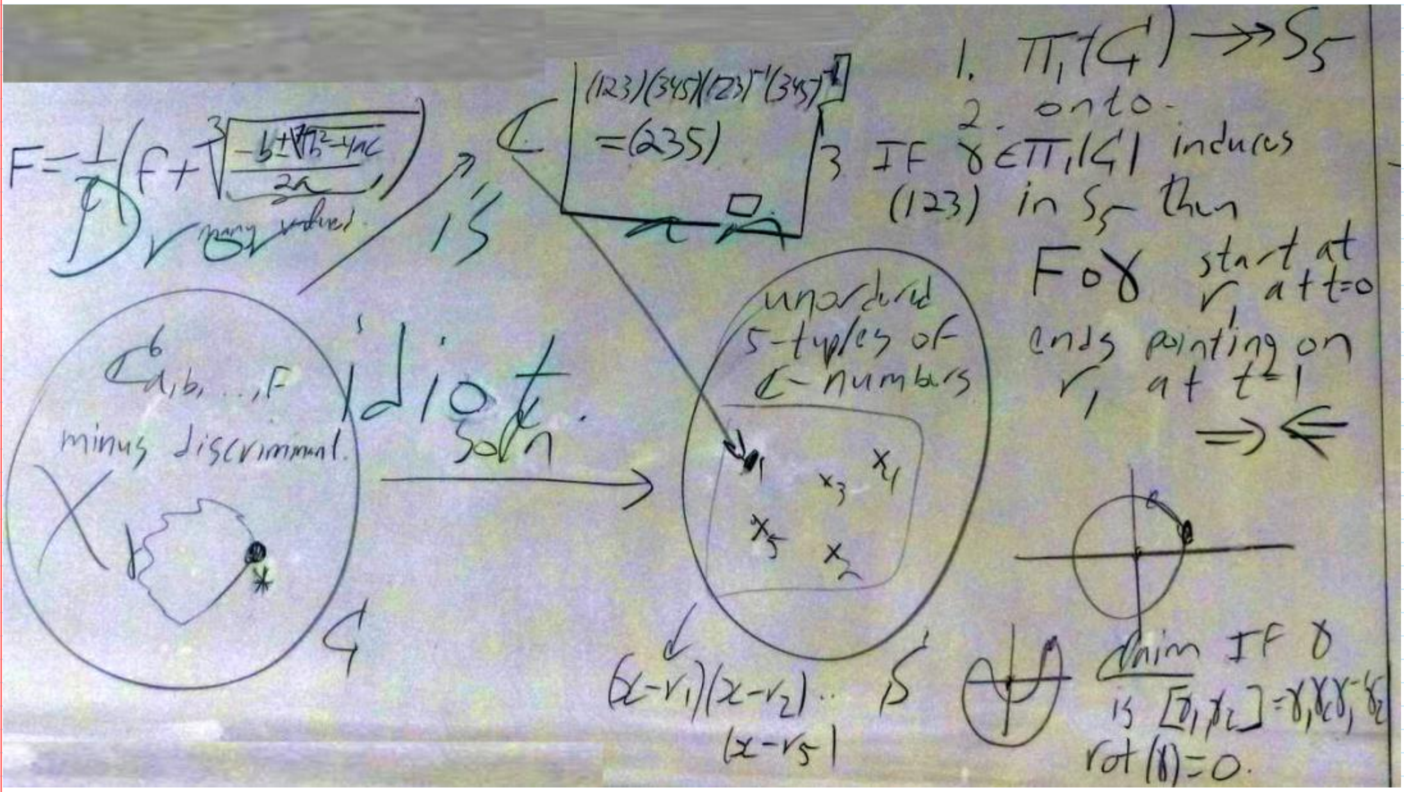
The Final: All is included, same style as term test & as previous years.

The key: Understand EVERYTHING.

Today: Not solving the quintic, more on JCF.

Tomorrow: Riddles session! Bahen 6183, 10 AM.

Following http://drorbn.net/dbnvp/AKT-140314.php:



Some JCF tricks

If $q = \gcd(a, b) = sa + tb$, the equality $\begin{pmatrix} s & t \\ -b/q & a/q \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} q \\ 0 \end{pmatrix}$ allows us to replace pairs of entries in the same column by their greatest common divisor (and a zero!), using invertible row operations. A similar trick works for rows.

If $1 = \gcd(a, b) = sa + tb$, the equality $\begin{pmatrix} sa & 1 \\ -tb & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & ab \end{pmatrix} \begin{pmatrix} a & -b \\ t & s \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ is an invertible row-column-operations proof of the isomorphism $\frac{R}{\langle a \rangle} \oplus \frac{R}{\langle b \rangle} \simeq \frac{R}{\langle ab \rangle}$.

A repeated application of the identity $\begin{pmatrix} p^{k-1} & -1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & p^k \end{pmatrix} \cdot \begin{pmatrix} 1 & p \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} p^{-1-k} & 0 \\ 1 & p \end{pmatrix}$ will bring a matrix like $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & p^4 \end{pmatrix}$ to the "Jordan" form of $\begin{pmatrix} p & 0 & 0 & 0 \\ 1 & p & 0 & 0 \\ 0 & 1 & p & 0 \\ 0 & 0 & 1 & p \end{pmatrix}$, using invertible row and column operations.

$$\langle x, y \rangle \Big/ \begin{matrix} y = 0 \\ p^k x = 0 \end{matrix} \cong \langle x, z \rangle \Big/ \begin{matrix} p^{k-1}x + z = 0 \\ pz = 0 \end{matrix}$$

$$\begin{aligned} y &\longmapsto p^{k-1}x + z \\ x &\longmapsto -x \\ -x &\longleftarrow\!\shortmid\; x \\ y + p^{k-1}x &\longleftarrow\!\shortmid\; z \end{aligned}$$