

Plan:

Mon May 27, pre-class. The Kauffman bracket and the Jones polynomial (with computations!), the Alexander polynomial, Khovanov homology.

Pasted from <<http://www.math.toronto.edu/~drorbn/Talks/Aarhus-1305/#plan>>

1. The Kauffman bracket & Jones polynomial, to invariance & $\chi - \chi = 1$
2. First program
3. Tangles & 2nd program (as in Knot Theory manual)
4. The Alexander polynomial as in Sussberg.

The collage contains several key elements:

- A Quick Introduction to Khovanov Homology:** A short text explaining the motivation for Khovanov homology as a categorification of the Jones polynomial.
- The Philosophy Course:** A section discussing the philosophical underpinnings of the mathematics, mentioning 'negative numbers' and 'the philosophy course'.
- Why Bother?:** A section with colorful diagrams and text explaining why Khovanov homology is interesting and useful.
- Grid of Diagrams:** A large grid showing the relationship between Khovanov homology and the Jones polynomial. The grid uses various symbols and arrows to represent the algebraic structure of the homology.

This collage includes:

- The Polynomial Alexander Invariant:** A section discussing the Alexander invariant and its relationship to the Jones polynomial.
- The Menus Business:** A section with various mathematical formulas and diagrams.
- References:** A list of references for the topics discussed in the notes.

5. Very basic Khovanov homology [Knot homology is not my goal, it is just an entertaining thing to keep in mind]

on board:
 This is not the "master class"
 of <http://www.math.toronto.edu/~drorbn/Talks/Aarhus-1305/>
 Today: Kauffman, Jones, Alex., Khovanov

$$\mathcal{P} \stackrel{?}{=} 0 \text{ mod}$$

$$1 \cup = 1, \chi = 1, \chi = 1$$

[KBDef] $\langle \emptyset \rangle = 1;$ $\langle \bigcirc L \rangle = (-A^2 - B^2)\langle L \rangle;$ $\langle \times \rangle = A\langle \succ \rangle + B\langle \rangle \langle \rangle;$

$$J(L) = (-A^3)^{w(L)} \frac{\langle L \rangle}{\langle \bigcirc \rangle} \Big|_{A \rightarrow q^{1/4}},$$

```
In[1]:= << KnotTheory`
```

```
Loading KnotTheory` version of February 5, 2013, 3:48:46.4762.  
Read more at http://katlas.org/wiki/KnotTheory.
```

```
In[2]:= TubePlot[TorusKnot[5, 4]]
```

```
Out[2]=
```



```
In[3]:= SetAttributes[P, Orderless];
```

```
KB0[pd_] := Expand[  
  Expand[Times@@pd /. X[a_, b_, c_, d_] => AP[a, d] P[b, c] + 1/AP[a, b] P[c, d]] //.  
  {P[a_, b_] P[b_, c_] => P[a, c], P[a_, b_]^2 => P[a, a], P[a_, a_] + -A^2 - 1/A^2}  
]
```

```
In[5]:= TorusKnot[5, 4] // PD // KB0 // Timing
```

```
Out[5]= {17.971315,  $\frac{1}{A^{23}} + \frac{1}{A^{19}} + \frac{1}{A^{15}} + \frac{1}{A^{11}} + \frac{1}{A^7} - A - A^5 - A^9$ }
```

```
In[6]:= SetAttributes[P, Orderless];
```

```
KB1[pd_PD] := KB1[pd, {}, 1];  
KB1[pd_PD, inside_, web_] := Module[  
  {pos = First[Ordering[Length[Complement[List@@#, inside]] & /@ pd]}],  
  pd[pos] /. X[a_, b_, c_, d_] => KB1[  
    Delete[pd, pos],  
    Union[inside, {a, b, c, d}],  
    Expand[  
      web * (AP[a, d] P[b, c] + 1/AP[a, b] P[c, d]) //.  
      {P[e_, f_] P[f_, g_] => P[e, g], P[e_, _]^2 => P[e, e], P[e_, e_] + -A^2 - 1/A^2}  
    ]  
  ];  
KB1[PD[], _, web_] := Expand[web]
```

```
In[10]:= TorusKnot[5, 4] // PD // KB1 // Timing
```

```
Out[10]= {0.046800,  $\frac{1}{A^{23}} + \frac{1}{A^{19}} + \frac{1}{A^{15}} + \frac{1}{A^{11}} + \frac{1}{A^7} - A - A^5 - A^9$ }
```