2.3. F.T.I of $v$-braids & $w$-braids.

2.3.1 The "classical/pictorial" approach.

1. F.T. for knots

2. semi-virtual xings,

$$V(X) := V(M) - V(N)$$

"Type $m \iff V(\geq m \text{ s.v.}) = 0$$

$$w = W_m(V) := V|_{\text{braids with exactly } m \text{ s.v.}}$$

$$G_m : M_{\text{sing}} / M_{\text{sing}}$$

= arrow diagrams

Claim. $w$ satisfies the "6T" relation.

pf.

$$V\left(\begin{array}{c} x \\ m-2 \text{ other} \\ \text{s.v.} \end{array}\right) = V\left(\begin{array}{c} x \\ m-2 \text{ other} \\ \text{s.v.} \end{array}\right)$$

Now write $0 \mapsto s_i + (s_i - s_i) = s_i + \overline{s_i}$
Claim. In the $w$ case, $W$ satisfies $OC \& \nabla^2$. In the $u$ case,...

**Definition.** $A^w_n, A^v_n, A^u_n$

**Definition 2.11** An expansion $\tau: v\beta_n \to A^u_n$

**Theorem.** An expansion exists iff every $W$ comes from a $V$.

### 2.3.2 Fi-T-I, The “algebraic” approach.

Start from a general group $G$,

define $FG, T, A_k(G)$, expansion

Claim. An invariant of $v$-braids is of type $m$ iff it vanishes on $T^m$.

Claim. $A^v_n \to A_k(v\beta_n)$

**Def.** Expansion $\tau: G \to A_k(G)$

**Claim** An $A^v_n$-expansion implies an expansion $\tau: A_k(G) \to A^v_n$.

Claim. $A_k$ is a functor.