HW#9 on web by midnight & Due: Tue Dec 4th

Basic properties of determinants:

0. \( \det(I_n) = 1 \)

1. \( \det(\begin{pmatrix} \circ & \circ & \cdots & \circ \\ \circ & r_i & \cdots & \circ \\ \vdots & \vdots & \ddots & \vdots \\ \circ & \circ & \cdots & r_j \end{pmatrix}) = \det(\begin{pmatrix} \circ & \circ & \cdots & \circ \\ \circ & r_i & \cdots & \circ \\ \vdots & \vdots & \ddots & \vdots \\ \circ & \circ & \cdots & r_j \end{pmatrix}) \)

2. \( \det(\begin{pmatrix} r_i \\ \vdots \\ r_j \end{pmatrix}) = c \det(\begin{pmatrix} r_i \\ \vdots \\ r_j \end{pmatrix}) \)

3. \( \det(\begin{pmatrix} r_i \\ -r_j + cr_i \\ \vdots \\ \vdots \end{pmatrix}) = \det(\begin{pmatrix} r_i \\ \vdots \\ \vdots \end{pmatrix}) \)

Example

\[ \det(\begin{pmatrix} x & 2 \\ 3 & 4 \end{pmatrix}) \]

Corollary: All that there is to know about determinants can be deduced from 0-3; also if \( \det \) satisfies 0-3, then \( \det' = \det \).

Thm: A is invertible iff \( \det(A) \neq 0 \)

Thm: If \( A = E_1 \cdots E_n \) is a product of elementary matrices, then \( \det A = \det(E_1) \cdet(E_2) \cdots \det(E_n) \)

Claim: For square matrices, \( AB \) invertible \( \Rightarrow AB \) and \( BA \) are invertible.

\[ (AB)^{-1} = B^{-1}A^{-1} \]

\( B(A)B^{-1} \) is a right inverse for \( A \), \( B \) for square matrices, if \( AC = I \) then also \( CA = I \).

Thm: \( \det AB = \det A \cdot \det B \)

Thm: \( \det A^T = \det A \)

Thm: Everything that's true for rows is also true for columns.

Skipped extra: 1. Other formulas for \( \det \) (row and column operations, permutations)

2. A \( \det \) formula for \( A^{-1} \) & Kronecker's law.
Recall the formula for $d^1$ & sketch the proof of the basic property:

$$|a_{i1}| = a_{i1} \quad \left| \begin{bmatrix} a_{i1} & \cdots & a_{in} \end{bmatrix} \right| = \sum_{j=1}^{n} (-1)^{i+j} a_{i1} \cdot \left| A_{ij} \right|$$

Then prove:

1. Linear in the first row.
2. Multilinear in the rows.
3. Vanishes if the first two rows are equal. \hspace{1cm} \text{Done}
4. Vanishes if two adjacent rows are equal
5. Switches sign if two adjacent rows are interchanged.
6. Switches sign whenever two rows are interchanged.
7. $E_{ij}^2$ & $E_{ij}^2$ behavior.