following 09-240 hour 16, taught by yael karshon:

• "t:v → w is linear"
• preserving 0.
• claim on cx+y.
• example: r^2 → r^2 by explicit formula.
• example: differentiation, multiplication by x.
• example: matrices and linear transformations on f^n.
• example: rotation (+ explicit formula).
• claim on differences and many-element sums.
• added 2012: call(v,w) is a vector space.
• composition of linear trans is a linear trans.
• composition is non-commutative. example: differentiation and multiplication by x.
• for a l.t., arbitrary values on a basis.
• "isomorphism".

Def v & w are isomorphic if:

ex. t: v → w and s: w → v

st. so t = iv & to s = iw

thm. if v/w are f.d. over f,

then dim v = dim w if v is isomorphic to w.

corollary. if dim v = n over f,

v is isomorphic to f^n.

"are two "mathematical structures are "isomorphic" if there's a bijection (1-1 & onto ones) between their elements which preserves all relevant relations. example: plastic chess is is to ivory chess, but not to checkers. example: the game of 15."

read along: sections 2.1-2.4
term test: thu oct 25 3-5pm, examination facility room ex 200 (east side of mccaul st., near college).
riddle along: 1 2 3 4 5 6 7 8 9

Two players alternate drawing cards from the above deck. The first player to have 3 cards that add up to 15, wins. Would you like to be the first to move or the second? (More on today's web, including a video link).
Fix a l.f. \( T : V \to W \)

**Def** \( N(T) = \ker T = \{ v :Tv = 0 \} \) "null space", "kernel"

\( R(T) = \text{im } T = \{ Tv : v \in V \} \) "range", "image"

**Prop/Def** \( N(T) \subset V \) is a subspace; \( \text{nullity } (T) = \dim N(T) \)

\( R(T) \subset W \) is a subspace; \( \text{rank } (T) = \dim R(T) \)

**Examples** \( O, I, D : \mathbb{P}_n(\mathbb{R}) \to \mathbb{P}_n(\mathbb{R}) \)

**Thm** "the dimension theorem", "the rank-nullity Thm"

Given \( T : V \to W \), \( \dim V = \text{rank } (T) + \text{nullity } (T) \)

**Prop** \( \{ z_i \} \) basis of \( N(T) \), extend to \( \{ z_i \} \cup \{ v_i \} \) a basis of \( V \),

claim \( w_i = T(v_i) \) are lin indep in \( W \) pf....

claim \( w_i \) span \( R(T) \) pf...