**Do not turn this page until instructed.**

# Math 240 Algebra I — Term Test

University of Toronto, October 25, 2012

**Solve 4 out of the 5 problems on the other side of this page.**
Each of the problems is worth 25 points.
You have an hour and 50 minutes.

**Notes.**

- No outside material other than stationary is allowed.
- **Neatness counts! Language counts!** The *ideal* written solution to a problem looks like a page from a textbook; neat and clean and made of complete and grammatical sentences. Definitely phrases like "there exists" or "for every" cannot be skipped. Lectures are mostly made of spoken words, and so the blackboard part of proofs given during lectures often omits or shortens key phrases. The ideal written solution to a problem does not do that.

**Good Luck!**

**Solve 4 of the following 5 problems.** Each of the problems is worth 25 points. You have an hour and 50 minutes. **Neatness counts! Language counts!**

**Problem 1.** Let $F$ be a field.

1. Prove that for any $a \in F$, we have $0 \cdot a = 0$.
2. Prove that if $a, b \in F$ and $ab = 0$, then either $a = 0$ or $b = 0$.

**Tip.** Don't start working! Read the whole exam first. You may wish to start with the questions that are easiest for you.

**Problem 2.**

1. In the field $\mathbb{C}$ of complex numbers, compute
$$4i(1+i) \qquad \text{and} \qquad \frac{4i}{1+i}.$$
(To be precise, "compute" means "write in the form $a + ib$, where $a, b \in \mathbb{R}$").
2. In the field $\mathbb{C}$ of complex numbers, find an element $z$ so that $z^2 = 2i$.
3. In the 11-element field $F_{11}$ of remainders modulo 11, find all solutions of the equation $x^2 = -2$.

**Problem 3.** Prove that if $W$ is a subspace of a finite-dimensional vector space $V$, then $W$ is also finite-dimensional.

**Problem 4.** Find a polynomial $f \in P_3(\mathbb{R})$ that satisfies $f(-1) = -3$, $f(0) = 0$, $f(1) = 1$, and $f(2) = 6$.

**Problem 5.** Let $V$ and $W$ be vector spaces over the same field $F$, and let $u$ be some element of $V$. Recall that $\mathcal{L}(V, W)$ denotes the vector space of all linear transformations $L\colon V \to W$.

1. Define a map $E\colon \mathcal{L}(V, W) \to W$ by setting $E(L) = L(u)$, for $L \in \mathcal{L}(V, W)$. Prove that $E$ is a linear transformation.
2. If in addition $V$ is of dimension 1 and $u \neq 0$, prove that $E$ is an isomorphism.

**Good Luck!**