& Agenda: Simplicity of $A_n$, group actions.

* Makeup class: Thursday at 9 AM?

Read Along?

* Go over handouts.

Definition. A $G$-set (left-$G$-set) $G \times X \rightarrow X$

s.t. $(g_1 g_2) x = g_1 (g_2 x)$, $e x = x$. Same as $\alpha: G \rightarrow S(X)$.

$G$-sets are a category.


2. Subgroups of $G$, under conjugation.

Examples: 1. $G/H$ when $H$ is not necessarily normal

Sub-example: $S_n / S_n^{-1}$, $G \sim S_n^{-1}$ iff

$G(n) \sim G(n)$. Let $\iota; n \sim n$, then

$G(n) \sim n^{-1} S_n^{-1}$. So $S_n / S_n^{-1}$ is $2(\ldots)$

2. If $X_1 , X_2$ are $G$-sets, then so is $X_1 \sqcup X_2$.

3. $S^2 \sim SO(3) / SO(2)$

Theorem. 1. Every $G$-set is a disjoint union of "transitive $G$-sets"

2. If $X$ is a transitive $G$-set and $x \in X$, then

$X \sim G / \text{stab}_x (x)$. (So $|X| \leq |G|)$

Theorem. If $X$ is a $G$-set and $x_i$ are representatives

of the orbits, then

$|X| = \sum \frac{|G|}{|\text{stab}_x (x_i)|}$
Example. If $G$ is a $p$-group, the Centre of $G$ is not empty.