René Abgrall. Selick 1.11, 2.1

HV 2 done.


Riddle Abgrall 2.

Agenda 12. Solvable, rings.

Claim \((\frac{Z}{2})^2 \times \frac{Z}{3}) \times \frac{Z}{3} \cong \frac{A_4}{2}

Solvable Groups. Def G is solvable if all quotients in its Jordan-Hölder series are Abelian.

Thm. If \(N \trianglelefteq G\), G is solvable iff \(N \trianglelefteq G/N\) are.

2. If \(H \trianglelefteq G\) and \(G\) is solvable, so is \(H\).

A \(\triangleleft B\) \(H \triangleleft A \triangleleft B\) \(\mbox{v} \) \(H \triangleleft B\) \(H/A \rightarrow B/A\) by \([b]_{B/A} \rightarrow [b]_A\).

Is injective.

Cor. If a group contains \(A_4\), \#4, it is not solvable.

Turn test line.

Rings.

Definition 2.1.1. A ring consists of a set \(R\) together with binary operations \(+, \cdot\) satisfying:

1. \((R, +)\) forms an abelian group.
2. \((a \cdot b) \cdot c = a \cdot (b \cdot c)\) \(\forall a, b, c \in R\),
3. \(\exists 1 \neq 0 \in R\) such that \(a \cdot 1 = 1 \cdot a = a\) \(\forall a \in R\), and
4. \(a \cdot (b + c) = a \cdot b + a \cdot c\) and \((a + b) \cdot c = a \cdot c + b \cdot c\) \(\forall a, b, c \in R\).

Also define:

Commutative Ring.

Examples. \((Z, \mathbb{Z}[x], \text{Mat}_n(R))\)

Morphisms:

1. \(Z \rightarrow \mathbb{Z}/n\)
2. \(R \rightarrow \mathbb{R}[x] \mbox{ as deg}\)
3. \(R \rightarrow \text{Mat}_n(R)\) as inv
4. \(\text{ev}_a : \mathbb{R}[x] \rightarrow R\) (if \(R\) is commutative)
\[
\begin{align*}
\text{im, subring, ker, ideal?} \\
\text{Q. Is every ideal a quotient.} \\
\text{Ans. Define } R/I. \\
\text{Good luck w/ term test!}
\end{align*}
\]