Non Commutative Gaussian Elimination @ MAT 1100

By Dror Bar-Natan

Amended from a similar notebook by Dror Bar-Natan and Itai Bar-Natan. The original version is at http://www.math.toronto.edu/~dorbn/Misc/SchreierSimsRubik/.

Pensieve Header: Non Commutative Gaussian Elimination @ MAT 1100 - as on handout + a printout of the filling table. See more at pensieve://2009-07/.

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Program 0

```plaintext
gs = {purple = P[18, 27, 36, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 45, 2, 20, 21, 22, 23, 24, 25, 26, 44, 1, 29, 30, 31, 32, 33, 34, 35, 43, 37, 38, 39, 40, 41, 42, 10, 19, 28, 52, 49, 46, 53, 50, 47, 54, 51, 48], white = P[1, 2, 3, 4, 5, 6, 16, 25, 34, 10, 11, 9, 15, 24, 33, 39, 17, 18, 19, 20, 8, 14, 23, 32, 38, 26, 27, 28, 29, 7, 13, 22, 31, 37, 35, 36, 12, 21, 30, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54], green = P[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 36, 48, 47, 46, 49, 42, 45, 38, 41, 44, 37, 40, 43, 30, 29, 28, 49, 50, 51, 52, 53, 54], blue = P[3, 6, 9, 2, 5, 8, 1, 4, 7, 54, 53, 52, 10, 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 18, 17, 16], red = P[13, 2, 3, 22, 5, 6, 31, 8, 9, 12, 21, 30, 37, 14, 15, 16, 17, 18, 11, 20, 29, 40, 23, 24, 25, 26, 27, 10, 19, 28, 43, 32, 33, 34, 35, 36, 46, 38, 39, 49, 41, 42, 52, 44, 45, 1, 47, 48, 4, 50, 51, 7, 53, 54], yellow = P[1, 2, 48, 4, 5, 51, 7, 8, 54, 10, 11, 12, 13, 14, 3, 18, 27, 36, 19, 20, 21, 22, 23, 6, 17, 26, 35, 28, 29, 30, 31, 32, 9, 16, 25, 34, 37, 38, 15, 40, 24, 24, 43, 44, 33, 46, 47, 39, 49, 50, 42, 52, 53, 45])};

($RecursionLimit = 2^16;
 n = 54;
P /: p_P ** P[a__] := p[{{a}}];
Inv[p_P] := P @@ Ordering[p];
Feed[P @@ Range[n]] := Null;
Feed[p_P] := Module[{i, j},
   For[i = 1, p[[i]] = i, ++i]; j = p[[i]];
   If[Head[s[i, j]] === P,
     Feed[Inv[s[i, j]] ** p],
   (*Else*) s[i, j] = p;
   Do[If[Head[s[k, l]] == P,
     Feed[s[i, j] ** s[k, l];
     Feed[s[k, l] ** s[i, j]]
   ],
   {k, n}, {l, n}]
   ]
   ];
(Feed[H]; Product[1 + Length[Select[Range[n], Head[s[i, H]] === P &]], {i, n}]) & @/gs

{4, 16, 159993501696000, 21119142223872000, 43252003274489856000, 43252003274489856000}
Images[___] := (i) \rightarrow \text{Join[Select[Range[n], Head[s[i, #]] == P &], ListPlot[Join@Table[i, #] & Images[i], {i, n}], AspectRatio \rightarrow 1]}

\(43252003274489856000 / (8! \times 3^8 \times 12! \times 2^{12})\)

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