September 30, hours 8-9: Hausdorff Spaces, Continuous Functions

September-27-10
5:03 PM

* class photo at break! HW1 expected! HW2 ~
* Read Along: Munkres 17–19. More OH?

<u>Definition</u> Limit pt: $x \in A' \Leftrightarrow x \in \overline{A - \{x\}}$,
iff every nbd of x contains a point of A other than x itself.

<u>Thm</u> $\overline{A} = A \cup A'$

Hausdorff spaces. Definition.

Properties. In a T_2 space X:

1. Points are closed.
2. $x \in A'$ iff every nbd contains infinitely many points of x.
3. A sequence converges to at most one limit.
4. Products of T_2 & subspaces of T_2 are T_2.

Continuous Functions. TFAE for $f: X \to Y$:

1. f is cont.
2. $f(\overline{A}) \subset \overline{f(A)}$
3. For every closed $B \subset Y$, $f^{-1}(B)$ is closed in X.
4. For every $x \in X$ & nbd V of $f(x)$, $\exists$ nbd U of x s.t. $f(U) \subset V$.

<u>pf</u> $1 \Rightarrow 2$ ✓

done line

$2 \Rightarrow 3$ Let $A = f^{-1}(B)$. $f(\overline{A}) \subset \overline{f(f^{-1}(B))} \subset \overline{B} \subset B$

So $\overline{A} \subset A$ so A is closed.

$3 \Leftrightarrow 1$ ✓

$1 \Leftrightarrow 4$ ✓

The Product Topology. On $\prod_{\alpha\in I} X_\alpha = \{f : I \to \bigcup X_\alpha : f(\alpha)\in X_\alpha\}$

1. Definition by properties

2. Basis.

The axiom of choice: "this is not empty".

The box topology. [on finite products, this is the same]

Example $\mathbb{R} \longmapsto \mathbb{R}^{\mathbb{N}}$ by $t \mapsto (t,t\ldots)$ is continuous in cyl but not in box [so box is strictly finer than cyl]

In both box & cyl:

1. The topology on $\prod A_\alpha \subset \prod X_\alpha$ as subspace is the same as ... as a product of subspaces.

2. If X_α is T_2 $\forall\alpha$, then $\prod X_\alpha$ is T_2.

3. $\overline{\prod A_\alpha} = \prod \overline{A_\alpha}$

Metrics & the metric topology.