September 27, hour 7: The Cantor Set, Closures

September-23-10
4:42 PM

- Start videotaping!
- I pledge to answer all general-interest wiki-asked questions before all general-interest email-asked questions.

Read Along: Munkres Sections 17–18.

Reminders 1. A basis for $X \times Y$ is $\{U \times V : \begin{matrix} U \subset X \\ V \subset Y \end{matrix} \text{ open}\}$

2. If $Y \subset X$, $\mathcal{T}_Y := \{U \cap Y : U \in \mathcal{T}_X\}$

3. F closed $\Leftrightarrow F^c$ is open.

on board.

Example: $C_0 = I = [0,1]$; C_n = From every interval in C_0, remove the open middle third. $C =$ "Cantor" $= \bigcap C_n$

Exercises 1. C has "length" 0.

2. C is uncountable, and has uncountably many "components".

3. $\exists F: [0,1] \to [0,1]$, cont., $F(0)=0$, $F(1)=1$, $F'(x)=0$ for all $x \notin C$. [Picture?]

Closed Sets. closed in closed is closed

open in open is open

closure & interior.

<u>Proposition</u> $x \in \bar{A}$ iff every (basic) nbd of x intersects A.

done line

<u>Definition</u> Limit pt: $x \in A' \Leftrightarrow x \in \overline{A - \{x\}}$, iff every nbd of x contains a point of A other than x itself.

<u>Thm</u> $\bar{A} = A \cup A'$

Hausdorff spaces. Definition.

Properties. In a T_2 space X:

1. Points are closed

2. $x \in A'$ iff every nbd contains infinitely many points of x.

3. A sequence converges to at most one limit.

4. Products of T_2 & subspaces of T_2 are T_2.

Continuous Functions. TFAE for $f: X \to Y$:

1. f is cont.
2. $f(\overline{A}) \subset \overline{f(A)}$
3. For every closed $B \subset Y$, $f^{-1}(B)$ is closed in X.
4. For every $x \in X$ & nbd V of $f(x)$, $\exists$ nbd U of x s.t. $f(U) \subset V$.

Pf $1 \Rightarrow 2$ ✓

$2 \Rightarrow 3$ Let $A = f^{-1}(B)$. $f(\overline{A}) \subset \overline{f(f^{-1}(B))} \subset \overline{B} \subset B$

So $\overline{A} \subset A$ so A is closed.

$3 \Leftrightarrow 1$ ✓

$1 \Leftrightarrow 4$ ✓