September 23, hour 5-6: Products, Subspaces, Closed Sets

September-20-10 3:34 PM

Read along: Munkres Sections 12-17. Videotaping?
HW1 is on web!

"The product topology"
Given X, Y topological spaces, we seek a topology on $X\times Y$ s.t. 1. $X\times Y \xrightarrow{\pi_X} X$, $X\times Y \xrightarrow{\pi_Y} Y$ are cont.

2. $f, g: Z \to X, Y$ cont. $\Rightarrow$ $f\times g: Z \to X\times Y$ is cont.

Thm Such a topology exists and is unique.

In future, may add $X \cong X\times\{y_0\}$ & $Y \cong \{x_0\}\times Y$.

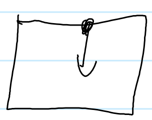
The Subspace Topology. Given a T.S. X and a subset $Y\subset X$, we seek a topology on Y s.t.

1. $i_Y: Y \hookrightarrow X$ is cont.
2. Given $f: Z \to Y \xhookrightarrow{i_Y} X$, if $i_Y\circ f$ is cont., then so is f.

Thm Such a topology exists and is unique.

Compatibilities. $\underbrace{\text{Sub \& Sub}}_{\text{prove}}$; $\underbrace{\text{Sub \& product}}_{\text{Leave as HW}}$; Sub & order (in the convex case)

Example The I^2_{dict} is different from $I^2_{in} \subset \mathbb{R}^2_{dict}$.



not open open

Closed Sets. Definition, 3 basic properties.

closed in a subspace — done line

closed in closed is closed

open in open is open

closure & interior.

Proposition $x \in \bar{A}$ iff every (basic) nbd of x intersects A.

Definition Limit pt: $x \in A' \Leftrightarrow x \in \overline{A - \{x\}}$, iff every nbd of x contains a point of A other than x itself.

Thm $\bar{A} = A \cup A'$

Hausdorff spaces. Definition.

Properties. In a T_2 space X:

1. Points are closed.
2. $x \in A'$ iff every nbd contains infinitely many points of x.
3. A sequence converges to at most one limit.
4. Products of T_2 & subspaces of T_2 are T_2.