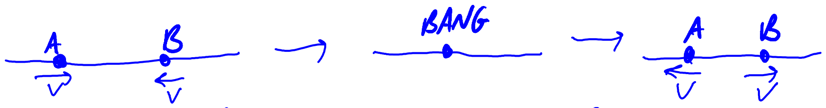
October 7, hours 11-12: Boxes and Cylinders

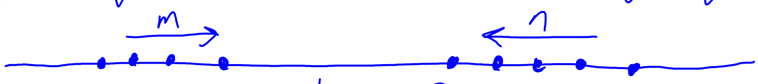
October-05-10

2:31 PM

HW1 returned (grades on Portal), HW2 due, HW3 on web by midnight.

Read along. Munkres 19, 20, 21. Additional OH: Thu 11:30–12:30, next week only 11:30–12:00





How many bangs?

The Product Topology. On $\prod_{\alpha\in I} X_\alpha = \{f : I \to \bigcup X_\alpha : f(\alpha)\in X_\alpha\}$

1. Definition by properties

   The axiom of choice: "this is not empty".

2. Basis.

The box topology. [on finite products, this is the same]

Example $\mathbb{R} \to \mathbb{R}^{\mathbb{N}}$ / $\mathbb{R}^{\omega}$ by $t \mapsto (t, t, \dots)$ is continuous in cyl but not in box [so box is strictly finer than cyl]

In both box & cyl:

1. The topology on $\prod A_\alpha \subset \prod X_\alpha$ as subspace is the same as ... as a product of subspaces.
2. If X_α is T_2 $\forall \alpha$, then $\prod X_\alpha$ is T_2.
3. $\overline{\prod A_\alpha} = \prod \overline{A_\alpha}$

Metrics & the metric topology.

* General defs.

done line

* Thm In a metric/metrizable space, closure = seq. closure.
* Thm $\mathbb{R}^{\mathbb{N}}_{box}$ is not metrizable. (No seq of positive seqs goes to $\bar{0}$)
* Thm $\mathbb{R}^{\mathbb{R}}_{cyl}$ is not metrizable
* A countable product of metrizable spaces is metrizable.

* A countable product of metrizable spaces is metrizable.
  * $\bar{d}(x,y) = \min(1, d(x,y))$