October 4, hour 10: Continuous Functions, Boxes and Cylinders

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3:33 PM

Read Along: Munkres 18, 19.

Continuous Functions. TFAE for $f: X \to Y$:

1. f is cont.
2. $f(\bar{A}) \subset \overline{f(A)}$
3. For every closed $B \subset Y$, $f^{-1}(B)$ is closed in X.
4. For every $x \in X$ & nbd V of $f(x)$, $\exists$ nbd U of x s.t. $f(U) \subset V$.

Pf $1 \Rightarrow 2$ ✓ $3 \Leftrightarrow 1$ ✓ $1 \Leftrightarrow 4$ ✓ done line

$2 \Rightarrow 3$ Let $A = f^{-1}(B)$. $f(\bar{A}) \subset \overline{f(f^{-1}(B))} \subset \bar{B} = B$

So $\bar{A} \subset A$ so A is closed.

on board

The Product Topology On $\prod_{\alpha \in I} X_\alpha = \{f: I \to \bigcup X_\alpha : f(\alpha) \in X_\alpha\}$

1. Definition by properties

The axiom of choice: "this is not empty".

not done 2. Basis.

The box topology. [on finite products, this is the same]

done line

Example $\mathbb{R} \longmapsto \mathbb{R}^{\mathbb{N}}$ by $t \mapsto (t, t, \dots)$ is continuous in cyl but not in box [so box is strictly finer than cyl]

In both box & cyl:

1. The topology on $\prod A_\alpha \subset \prod X_\alpha$ as subspace is the same as ... as a product of subspaces.
2. If X_α is T_2 $\forall \alpha$, then $\prod X_\alpha$ is T_2.
3. $\overline{\prod A_\alpha} = \prod \bar{A}_\alpha$

Metrics & the metric topology.