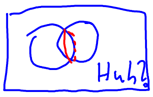
October 25, hour 18: Compactness

October-25-10
8:54 AM

HW. Return HW/3.

The TT: At ss1087!

Read Along. Munkres 26, 27

Riddle Along. 

Def. Cover, open cover, compact.

Thm. A continuous function on (to $\mathbb{R}$) a compact set is bounded.

PF 1 Local to global.

PF 2 Sneaky $- \; X = \bigcup_{n=1}^{\infty} f^{-1}(-n,n)$.

Example. A finite set is compact.

Thm. $[0,1]$ is compact.

Proof. Let $\mathcal{U}$ be an open cover of $[0,1]$.

Let $G = \left\{ g \in [0,1] : \begin{array}{l} \text{a finite subset} \\ \text{of } \mathcal{U} \text{ covers } [0,g] \end{array} \right\}$ Wish: $1 \in G$.

G is non-empty and bounded, so $g_0 = \sup(G)$ exists.

Step 1. $g_0 > 0$.

Step 2. $g_0 = 1$.

Step 3. $1 \in G$.

done line

Thm. A closed subset of a compact space is compact.

Thm. A compact subset of a T_2 space is closed.

Corollary. A subset of $\mathbb{R}$ is compact iff it is closed and bounded.