October 14, hours 13-14: Metrics, Boxes and Cylinders

October-12-10
4:35 PM

Read Along: Munkres 20, 21, 23, 24

Metrics & the metric topology. } on board.

* General defs.

* Thm In a metric/metrizable space, closure = seq. closure.

* Thm $\mathbb{R}^{\mathbb{N}}_{box}$ is not metrizable. (No seq of positive seqs goes to $\bar{0}$)

* Thm $\mathbb{R}^{\mathbb{R}}_{cyl}$ is not metrizable

* A countable product of metrizable spaces is metrizable.

  * $\bar{d}(x,y) = \min(1, d(x,y))$

done line

Connectedness. Separation, connectedness, clopen sets.

The I.V.T. If X is connected, $f: X \to \mathbb{R}$ cont.,

$$f(x_0) < 0,\ f(x_1) > 0 \Rightarrow \exists x \text{ s.t. } f(x) = 0.$$

Theorem $I = [0,1]$ is connected.

Proof. Assume $0 \in A \subset I$ is clopen. Let

$$G = \{x : [0,x] \subset A\} \qquad g = \sup G$$

1. $g > 0$ 2. $g \not< 1$ 3. $1 \in G$.

Theorem. If $A_\alpha \subset X$ are connected, $\bigcap A_\alpha \neq \emptyset$, then $\bigcup A_\alpha$ is connected.

Theorem. $A \subset \mathbb{R}$ is connected iff it is an interval, or a ray, or the whole thing. [I.e., if it is "convex"]

Theorem. If A is connected & $A \subset B \subset \bar{A}$, B is too.

Pf Assume C is clopen in B, $C \cap A \neq \emptyset$. Then

$C \supset A$ so $cl_X C \supset \bar{A} \supset B$, so $cl_X C \cap B \supset B$,

so $cl_B C \supset B$, so $C \supset B$.

Theorem If $\forall \alpha$ X_α is connected, then $\prod X_\alpha$

Theorem. If $\forall \alpha$ X_α is connected, then $\prod X_\alpha$ is connected.

Example. $\mathbb{R}^\omega = \left\{\begin{matrix}\text{bndd}\\ \text{seqs}\end{matrix}\right\} \cup \left\{\begin{matrix}\text{unbndd}\\ \text{seqs.}\end{matrix}\right\}$ is a box-separation.