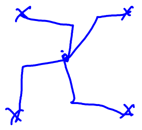
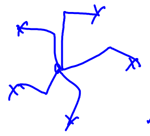
November 29, hour 32: Compactness in metric spaces

November-28-10
11:14 AM

Riddle Along. $\mathcal{E}($  $) = ?$ $\mathcal{E}($  $) = ?$

HW. HW8 (last!) on web by midnight. | OH. Distribute Dec Schedule.

Goal: Compactness in Metric Spaces [Munkres 43,45]

Theorem. The Following are Equivalent for a Metric X:

1. X is compact.
2. X is "limit-point-compact".
3. X is "sequentially compact."
4. X is "totally bounded" & "satisfies Lebesgue's Lemma".
5. X is totally bounded & "complete".

Quoted phrases need to be defined!

From Math 300 Topology of Nov 1, 2004 (2 hours):

http://katlas.math.toronto.edu/drorbn/notebooks/show?page=0405-1300_P5120166.jpg

Pf: $1 \Rightarrow 2$, $2 \Rightarrow 3$ done

Define "totally bounded"

State "Lebesgue Lemma"

{Aside & pf: If X is metric & compact & $f: X \to Y$ is continuous, then f is uniformly cont. [what is the #/URL]}

Pf of $3 \Rightarrow 4$: 1. X is totally bounded (by contradiction)

2. Lemma: Every cont. function on X attains its max.

Pf of Lemma

Thm from Lemma. $f(x) = \sup\{r : \exists U \in \mathcal{U}\ B_r(x) \subset U\}$

not done

Pf of $4 \Rightarrow 1$: Easy

Define "complete" (Every Cauchy seq. converges)

Pf of $3 \Rightarrow 5$

Pf of $5 \Rightarrow 3$

Aside: Every metric space has a "completion" - a complete metric space in which X is isometrically embedded densely.