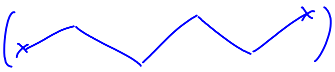
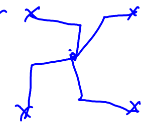
November 25, hours 30-31: Tietze

November-24-10
2:32 PM

**Riddle Along.** $\mathcal{E}($  $)=?$ $\mathcal{E}($  $)=?$

$\mathcal{E}($  $)=?$

**Read Along.** Munkres 35, 43, 45

**HW.** HW6 returned, HW7 due, HW8 assigned.

**Goal.** Tietze, compactness in metric spaces.

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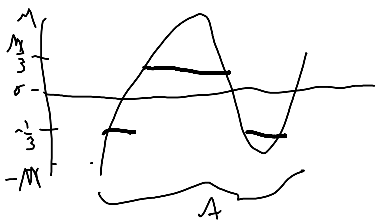
**Tietze's Theorem.** If $X$ is $T_4$, $A \subset X$ closed, and $f: A \to \mathbb{R}$ is cont., then there is a cont. extension $\tilde{F}$ of $f$ to all of $X$. (So $\tilde{F}|_A = f$)

**Remark.** Tietze $\Rightarrow$ Urysohn.

**Pf. of Tietze.** Assume first that $f$ is bndd.




__Lemma__ Suppose $\tilde{F}_0 : X \to \mathbb{R}$ is such that $|f - \tilde{F}_0| \leq \epsilon$ on $A$; then $\exists\ \tilde{F}_1 : X \to A$ s.t. $|\tilde{F}_0 - \tilde{F}_1| \leq \epsilon$ everywhere & $|f - \tilde{F}_1| \leq \frac{2\epsilon}{3}$ on $A$.

So construct $\tilde{F}_0 = 0, \tilde{F}_1, \tilde{F}_2 \dots$ s.t.

$$|f - \tilde{F}_n| < \left(\frac{2}{3}\right)^n M \quad \text{on } A$$

$$|\tilde{F}_n - \tilde{F}_{n+1}| < \left(\frac{2}{3}\right)^{n-1} \quad \text{on } X$$

Let $\tilde{F}(x) = \lim_{n\to\infty} \tilde{F}_n(x)$ (limit exists as $\tilde{F}_n(x)$ is cauchy)

__Lemma__ A uniformly convergent sequence of cont. functions converges to a cont. limit.

__Proof__ ....

Now do the unbounded case.....