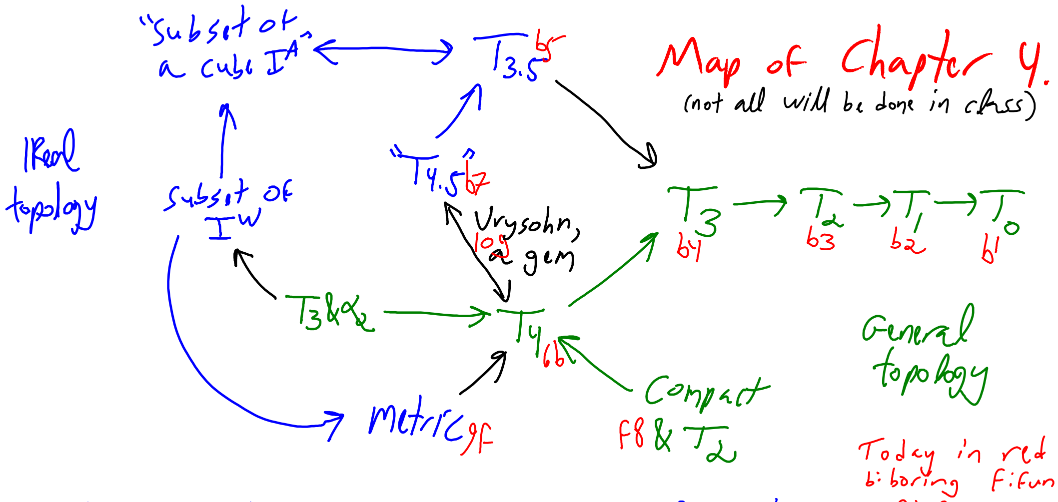
November 18, hours 27-28: Normal spaces

November-17-10
3:49 PM




**HW.** HW6 due, HW7 on web by midnight.

**Read Along.** Munkres 31-33.

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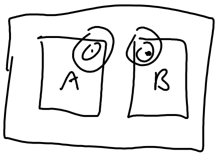
**Do B1-B7:** $T_0 \leftarrow T_1 \leftarrow T_2 \leftarrow T_3 \leftarrow T_{3.5} \leftarrow T_4 \overset{g}{\leftarrow} T_{4.5}$

**F8: Theorem.** $X$ compact $T_2 \Rightarrow X$ normal.

**Pf.** The usual.

**F9: Theorem.** $X$ metric $\Rightarrow X$ normal.

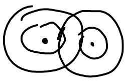
**Pf.**



For $a \in A$ find $\epsilon_a$ s.t. $B(a,\epsilon_a) \subset B^c$; set $U = \bigcup B(a, \frac{\epsilon_a}{2})$

For $b \in B$ find $\epsilon_b$ s.t. $B(b,\epsilon_b) \subset A^c$; $V = \bigcup B(b, \epsilon_b/2)$

$U$ & $V$ are disjoint: $B\left(a, \frac{\epsilon_a}{2}\right) \cap B\left(b, \frac{\epsilon_b}{2}\right) = \emptyset$

 . . . .

**G10: Urysohn's Lemma.** $T4 \Rightarrow T4.5$: If $X$ is normal & $A, B$ are disjoint & closed, then $\exists f: X \to I$ cont. s.t. $f(A)=0$, $f(B)=1$.

**Proof.** For every rational $0 \le q \le 1$ we'll construct an open $U_q$, s.t. $q < r \Rightarrow A \subset U_q \subset \overline{U_q} \subset U_r \subset B^c$

1. For $q=1$, take $U_1 = B^c$

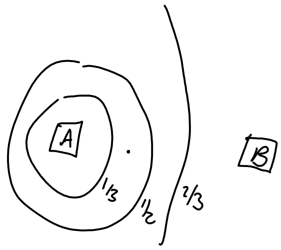
2. For $q=0$, find a separation $A \subset U_0$, $B \subset V_0$, $U_0 \cap V_0 = \emptyset$, and take $U_0$

3. Order the rationals $q_1 = 1$, $q_2 = 0$, $q_3$, $q_4$, ..., assume

$U_{q_1}, \dots U_{q_{k-1}}$ were found. Let $a$ be the biggest of $q_1 \dots q_{k-1}$ smaller the $q_k$, $b$ the smallest of $q_1 \dots q_{k-1}$ larger than $q_k$. Need $\overline{U_a} \subset U_k \subset \overline{U_k} \subset U_b$; find it by separating $\overline{U_a}$ & $U_b^c$.

4. Set $U_{<0} = \emptyset$, $U_{>1} = X$ and

$$f(x) = \inf\{q : x \in U_q\}$$




$f(A)=0$, $f(B)=1$,

5. Note: $x \in U_q \Rightarrow f(x) \le q$; $f(x) < q \Rightarrow x \in U_q$

$x \in U_q^c \Rightarrow f(x) \ge q$; $f(x) > q \Rightarrow x \in \overline{U_q}^c$

6. $f$ is cont; indeed, given $x$ & $\epsilon > 0$, find $a, b \in \mathbb{Q}$ with $x - \epsilon < a < x < b < x + \epsilon$, and then $\overline{U_a}^c \cap U_b = V$ is a nbd of $x$ for which $f(V) \subset (x-\epsilon, x+\epsilon)$.

All done!