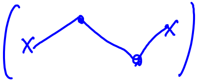
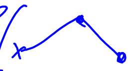
November 15, hour 26: Stone-Cech, Lims

November-15-10
9:44 AM

Goal. A taste of stone-Cech [still hardest]

HW.

Riddle Along. $\mathcal{C}($  $)=?$ $\mathcal{C}($  $)=?$ $\mathcal{C}($  $)=?$

Reminders. $A=\left\{\begin{matrix}\text{bounded}\\ \text{sequences}\end{matrix}\right\}=\{(a_j)\}\subset\mathbb{R}^{\mathbb{N}}$

$R_{(a_j)}:=[\inf(a_j),\sup(a_j)]\subset\mathbb{R}$ $\quad C=\prod_{a\in A} R_a$ is compact!

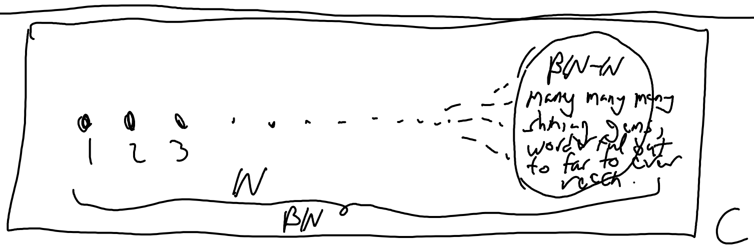
$\phi:\mathbb{N}\hookrightarrow C$ by $\phi(k)_a=a_k$

$\beta\mathbb{N}:=\overline{\phi(\mathbb{N})}\subset C$ is compact & has $\mathbb{N}$ as a dense subset.

<u>Claim</u> Every bndd $b:\mathbb{N}\to\mathbb{R}$ has a unique cont. extension $\tilde{b}$ to $\beta\mathbb{N}$

$$\begin{tikzcd} \mathbb{N} \arrow[r, hook, "\phi"] \arrow[dr, "\text{given } b"'] & \beta\mathbb{N} \arrow[d, "\exists!\tilde{b}"] \\ & \mathbb{R} \end{tikzcd}$$

<u>proof</u> set $\tilde{b}=\pi_b|_{\beta\mathbb{N}}$.



<u>Claim</u> If $\mu\in\beta\mathbb{N}-\mathbb{N}$ and $b=(b_j)$ is a seq. whose limit $\lim_{j\to\infty} b_j$ exists, then $\tilde{b}(\mu)=\lim_{j\to\infty} b_j$.

<u>Pf</u> Enough to know that $\mu\in\overline{[n,\infty)}$, for any n.

In a T_2 space, $\mu\in\overline{D}-D$ iff every nbd of μ contains infinitely many elements of D.

Now fix μ and define $\mathrm{Lim}_\mu(b_j)=\tilde{b}(\mu)$. Then Lim is a "generalized limit".

This contradicts the principle of diminishing values.

Thm. (Prove not) $\beta\mathbb{N}\longleftrightarrow\left\{\begin{matrix}\text{the set of all finitely}\\ \text{additive } \{0,1\}\text{-valued measures}\\ \text{on } \mathbb{N}\end{matrix}\right\}$

show $\longrightarrow$, discuss.