December 6, hour 35: Baire Spaces

December-04-10
8:05 AM

Discussion of the final.

Discussion of Dec 8. (Check about using a projector)

Read Along: Munkres Sections 48, 49

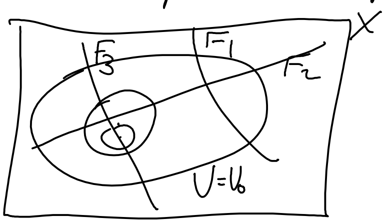
Def. A Baire space is a space X s.t. any countable union of closed sets w/ empty interior ("thin") has empty interior ("not chunky")

(in complements! Every countable intersection of open dense sets is dense) (counter example: $\mathbb{Q}$)

Thm. A complete metric space is Baire.

Thm. A compact T_2 space is Baire. (do not prove)

Pf (Show that $U \not\subset \bigcup \text{int} F_k$)



start with $U_0 = U$, construct U_n open s.t. $\overline{U_n} \subset U_{n-1} \setminus F_n \subset U \setminus \bigcup_{k \le n} F_k$

Example. "Most cont. functions on I are nowhere diff'able."

$X = (\mathcal{C}([0,1]), d_\infty)$ — a complete metric space.

$$U_n = \left\{ f : \begin{array}{l} \exists \text{ partition } 0 = x_0 < x_1 < \dots < x_p = 1 \\ \forall i \ |x_{i+1} - x_i| < \frac{1}{n} \ \& \ \left| \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i} \right| > n \end{array} \right\}$$

Claim I U_n is open & dense

Claim II $g \in \bigcap U_n \Rightarrow g$ is nowhere diff'able.

All done, if dense at end.