December 2, hours 33-34: Compactness in metric spaces

November-29-10
5:18 PM

Course evaluations today! HW8 returned at break.

Continue as on Nov 29.

done line

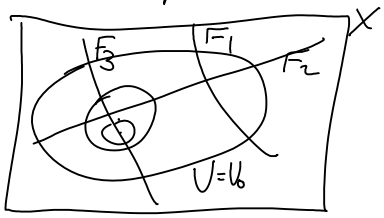
If time: Munkres section 48:

Def. A Baire space is a space X s.t. any countable union of closed sets w/ empty interior has empty interior.
("thin" / "not chunky")

(in complements! Every countable intersection of open dense sets is dense) (counter example: $\mathbb{Q}$)

Thm. A compact T_2 space is Baire.

Pf (show that $U \subseteq \operatorname{int} U$)



Start with $U_0 = U$, construct U_n open s.t. $\overline{U_n} \subseteq U_{n-1} \setminus F_n \subseteq U \setminus \bigcup_{k \le n} F_k$

Thm. A compact T_2 space with no isolated points is uncountable.

Thm. A complete metric space is Baire.

Thm. An open subset of a Baire space is Baire.