Using Gaussian Elimination

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9:41 AM

$$M \cong R^k \oplus R/\langle p_1^{s_1}\rangle \oplus R/\langle p_2^{s_2}\rangle \oplus \cdots \oplus R/\langle p_n^{s_n}\rangle$$

See also http://mathoverflow.net/questions/12009/is-there-a-slick-proof-of-the-classification-of-finitely-generated-abelian-groups, quoted on the next page.

Sketch.

1. $R^n \xrightarrow{A} R^m \to M \to 0$ $\quad n \le m$

$\mathcal{F}$ as a submodule of a free module is free and of a lower rank.

$\mathcal{F}$ as M is f.g.

Need: submods of a free mod.

2. Row & column reduction over a PID $\Rightarrow$ W.L.O.G A is diagonal.

need to be done!

3. split

$R/\langle ab\rangle = R/\langle a\rangle \oplus R/\langle b\rangle$ if $\gcd(a,b)=1$

... reach $(p_i^{s_i})$

consolidate

$\frac{R}{\langle a\rangle} \oplus \frac{R}{\langle b\rangle} = \frac{R}{\langle lcm(a,b)\rangle} \oplus \frac{R}{\langle \gcd(a,b)\rangle}$

need slick proofs.

... reach $a_1, \ldots, a_n$ s.t. $a_n \mid a_{n-1} \mid \cdots \mid a_1$

4. Uniqueness of torsion part:

$$\frac{R}{\langle a\rangle} \otimes \frac{R}{\langle b\rangle} = \frac{R}{\langle a,b\rangle} = \frac{R}{\langle \gcd(a,b)\rangle}$$

Need $\left(R/\langle p^s\rangle\right)^k$ determines k.

5. Uniqueness of the free part Needs to be done.