September 14, hours 1-2 - Non Commutative Gaussian Elimination

September-11-10

12:53 PM

MAT 1100 Core Algebra. To do. 1. print "About". ~~2. Print NCGE.~~ ~~3. Video tape~~?

DROR BAR-NATAN [website: search]

I don't know core algebra!

on book

Goal: Within your lifetime, understand $G = \langle g_1 \ldots g_m \rangle \subset S_n$:

1. $|G| = ?$ 2. $\sigma \in G?$ 3. $\sigma = w(g_1, \ldots g_m)$ 4. random σ

Two pre-requisites 1. Groups, S_n, silly uniquenesses, cancellation, $(ab)^{-1} = b^{-1}a^{-1}$, subgroups, the subgroup generated by σ_α.

2. Row reduction for real.

$\boxed{f \cdot g = f \circ g}$

Algorithm as in handout.

Claim 1 Every σ_{ij} in T is in G.

Claim 2 Anything fed to T is now a monotone product $\sigma_{1j_1}\sigma_{2j_2}\sigma_{3j_3}\cdots$ $j_i \geq i$

Claim 3 If two monotone products are equal,

$$\sigma_{1j_1} \cdots \sigma_{nj_n} = \sigma_{1j_1'} \cdots \sigma_{nj_n'}$$

then all the indices are equal, $\forall i\ j_i = j_i'$.

Claim 4 Let $M_k = \left\{ \begin{matrix} \text{monotone products} \\ \text{beginning with } k \end{matrix} \right\} = \{\sigma_{kj_k} \cdots \sigma_{nj_n}\}$, then for every k, $M_k \cdot M_k \subset M_k$ (and so each M_k is a subgroup of S_n.

done line

Proof Clearly $M_n M_n \subset M_n$. Now assume that $M_5 M_5 \subset M_5$ and show that $M_4 M_4 \subset M_4$. Start with $\sigma_{8,j} M_4 \subset M_4$:

$$\sigma_{8,j}(\sigma_{4,j_4} M_5) \stackrel{1}{=} (\sigma_{8,j}\sigma_{4,j_4}) M_5 \stackrel{2}{\subset} M_4 M_5$$

$$\stackrel{3}{=} \sigma_{4,j_4}(M_5 M_5) \stackrel{4}{\subset} \sigma_{4,j_4} M_5 \subset M_4$$

Claim 5. $M_1 = G$ and we have achieved all of

our goals [except there is a hidden problem].
→ then do goals 1, 2, 3, 4 and the 0: "in our lifetime".

<u>Example</u> $\sigma_1 = (123)$ $\sigma_2 = (12)(34)$, in S_4
$ \| \|$
$ 2314 2143$

11 I			
12 $\sigma_1 = 2314$ 1	22 I		
13 $\sigma_{12}^2 = 3124$ 2	23 $\sigma_{12}^{-1}\sigma_2 = 1342$ 3	33 I	
14 $\sigma_{23}\sigma_{13} = 4132$ 5	24 $\sigma_{13}^{-1}\sigma_{23}\sigma_{12} = 1423$ 4	34	44 I

Feed $\sigma_1 = 2314$ ... Fed @ σ_{12}

Feed $\sigma_{12}^2 = 3124$ ... Fed @ σ_{13}

Feed $\sigma_2 = 2143$ ... Feed $\sigma_{12}^{-1}\sigma_2 = 1342$ .. Fed @ σ_{23}

Feed $\sigma_{12}\sigma_{23} = 2143$ ... Feed $\sigma_{12}^{-1}\sigma_{12}\sigma_{23} = \sigma_{23}$ ....

No point feeding $\sigma_{ij}\sigma_{kl}$ if $i<k$!

Feed $\sigma_{23}\sigma_{12} = 3412$ ... Feed $\sigma_{13}^{-1}\sigma_{23}\sigma_{12} = 1423$ ... to σ_{24}

Feed $\sigma_{23}\sigma_{13} = 4132$ .. to σ_{14}

Feed $\sigma_{24}\sigma_{12} = 4213$ ... Feed $\sigma_{14}^{-1}\sigma_{24}\sigma_{12} = 1423$ ... drop.

$\Rightarrow |G| = 4 \cdot 3 \cdot 1 \cdot 1 = 12.$ Is $4123 \in G$?

write 2431 in terms of $\sigma_{1,2}$.

* Go over the "about" handout.