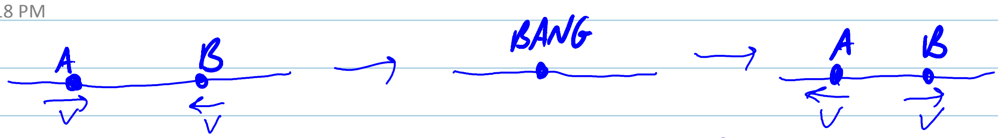

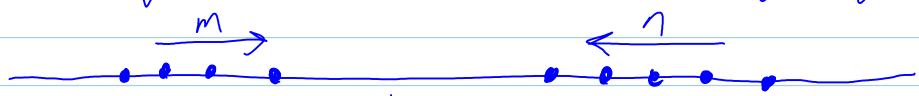

How many bangs?

Read Along. Selick notes 1.7, D&F 4.5.

Theorem. 1. Every G-set is a disjoint union of "transitive G-sets"

2. If X is a transitive G set and $x \in X$, then $X \cong G/\mathrm{stab}_X(x)$. (So $|X| \mid |G|$)

More Examples. 1. G acting on $\{$Subgroups of $G\}$

2. G/H when H is not-necessarily normal

Sub-example: S_n/S_{n-1} $\quad \sigma S_{n-1} = \sigma' S_{n-1}$ iff $\sigma(n) = \sigma'(n)$. Let $\tau_i(n) = n$, then $\sigma \tau_i S_{n-1} = \tau_{\sigma i} S_{n-1}$. So S_n/S_{n-1} is $\{1 \dots n\} \dots$

3. $S^2 = SO(3)/SO(2)$

Proof of 2. ......

Theorem. If X is a G set and x_i are representatives of the orbits, then

$$|X| = \sum_i \frac{|G|}{|\mathrm{stab}_X(x_i)|}$$

Example. If G is a p-group, the centre of G is not empty.

done line

done line

THE SYLOW THEOREMS.

$|G| = p^\alpha m$, p prime, $p \nmid m$; $\mathrm{Syl}_p(G) := \{P < G : |P| = p^\alpha\}$ are "Sylow p-subgroups of G". A "p-subgroup" in general, is any subgroup of G of order a power of p.

Sylow I $\mathrm{Syl}_p(G) \neq \emptyset$.

Proof. By induction on $|G|$, if G has a normal subgroup of order p (or p^β) or if G has a subgroup of order divisible by p^α, we are done. The existance of one of the said types follows from the class equation:

$$|G| = \underbrace{|Z(G)|}_{\text{the centre of } G} + \sum_i (G : \underbrace{C_G(y_i)}_{\text{the centralizer of } y_i \text{ in } G})$$

Where $\{y_i\}$ are representatives from the non-central conjugacy classes of G. $\square$