October 21, hour 18: Braids, Orders pq and 12, Solvable groups

Read Along: Schick 18.1.10, 1.11.

Riddle Along: \( \forall x \in \mathbb{E}, \exists \alpha \exists \beta \in \mathbb{Q} \) what do these solve?

Aside. Go over MaxPermOrder.

Big Example. \( B_n = \mathbf{F}_{11}((\mathbb{C}^2 - \text{Sing}h) / S_n) = \frac{\mathbb{C}^2}{\mathbb{C}^2 \langle e^i, e^j \rangle} \)

\[ B_n = \langle e^i, \ldots, e^{n-1} : \begin{align*}
    0 & \quad 0 = 1, \\
    0 & \quad 0 = 1, \\
    1 & \quad 1 = 1, \\
    1 & \quad 1 = 1
\end{align*} \]

in a sense on free groups generates all relations.

TT: \( B_n \rightarrow S_n \) \( PB_n = \ker T \)

PBn \cap Bn yet not \( B_n = S_n \times PB_n \)

\( \rho : PB_n \rightarrow PB_n^{-1} \) \( \ker \rho = F_{n-1} \) and \( \rho \) \( PB_n = PB_n^{-1} \times F_{n-1} = (\langle \ldots (F_1 \times F_2) \times F_3 \ldots \times F_{n-2} \rangle \times F_n) \times F_{n-1} \)

Two reasons why I like this one:

1. knotted $\rho$'s.
2. Borromean.

Groups of order 21. \( \mathbb{Z}/21, \mathbb{Z}/4 \times \mathbb{Z}/3 = \langle x, y \rangle \times \langle x, y \rangle \)

\[ \text{Aut}(\mathbb{Z}/3) = \mathbb{Z}/6 = \langle \phi_3 \rangle ; \phi_3(x) = x^3, x^3 = x \text{ or } x^2 = x \]

(iso: if \( x^3 = x^2 \) & \( y = y^2 \) then \( x^5 = x^4 \))

Groups of order 12. If \( 161 = 12, P_4 = \mathbb{Z}/4 \) or \( \mathbb{Z}/2 \times \mathbb{Z}/3 \), \( P_4 = \mathbb{Z}/3 \) and at least one of these is normal, for there’s not enough room for \( 4 P_3 \) & \( 3 P_4 \)’s. So \( G \) is a semi-direct product.

\[ \mathbb{Z}/12, \mathbb{Z}/4 \times \mathbb{Z}/3, \mathbb{A}_4, \text{D}_4, \mathbb{Z}/3 \times \mathbb{Z}/4 \]

The most fun case is \( \mathbb{Z}/3 \times (\mathbb{A}_4)^2 \) giving \( A_4 \).

Solvable Groups. Def \( G \) is solvable if all quotients in its Jordan-Hölder series are Abelian.

Thm 1. If \( N \triangleleft G \), \( G \) is solvable iff \( N \times G / N \) are.

2. If \( HK \triangleleft G \) and \( G \) is solvable, so is \( H \).

\[ A \cap B \triangleleft A \triangleleft B \vee \frac{H \cap B}{H \cap A} \rightarrow \frac{B}{A} \text{ by } [b]_{H \cap A} \rightarrow [b]_A \]

is injective.