November 25 Scratch

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9:44 PM

$x^2 = (1+y)(1-y)$

$R = \mathbb{Q}[x,y]/\langle x^2+y^2-1\rangle \cong \mathbb{Q}[x,y]/\langle x^2+y^2+2y\rangle$

claim x is irred in the above.

$z = x+iy$
$z^{-1} = x-iy$

$R \longrightarrow \mathbb{C}[z,z^{-1}]$ by $x \mapsto \frac{z+z^{-1}}{2}$ $y \mapsto \frac{z-z^{-1}}{2i}$

in here $\frac{z+z^{-1}}{2}$ is $\frac{1}{2}(z-i)(1+i/z) = \frac{1}{2}(x+iy-i)(1+y+ix)$

$= -\frac{i}{2} + x + \frac{1}{2}(x+iy)(y+ix)$

$= x$

$$\boxed{(z+z^{-1})^2 = (2-iz+iz^{-1})(2+iz-iz^{-1})}$$

$= 4 + (z-z^{-1})^2 = z^2+2+z^{-2}$