Lims

November-03-10
9:13 PM

Continued in 11-1100/Lims.

$S = \{\text{bndd seq's in } \mathbb{R}\}$ $\quad I = \{(a_n) : a_n \neq 0 \text{ for finitely many } n\text{'s}\}$

J - a maximal ideal containing I.

$\mathrm{Lim} : S \longrightarrow S/J \overset{?}{=} \mathbb{R}$

<u>Claim</u> $S = \{\alpha \bar{1} + (a_n) : \alpha \in \mathbb{R},\ (a_n) \in J\}$

$(b_n) \notin S \Rightarrow c_n \cdot b_n + a_n = 1 \quad b_n = \frac{1-a_n}{c_n}$

$\mathrm{Lim} : S \to S/J =$

~~<u>Claim</u> $0 \leq (a_n) \leq (b_n) \in J \Rightarrow (a_n) \in J$.~~ $\quad c_n = \begin{cases} \frac{b_n}{a_n} & a_n \neq 0 \\ 1 & a_n = 0 \end{cases}$

~~Pf assume not, then $\exists c_n$ s.t. $c_n a_n - b \in J$ $\quad c_n a_n \in J$~~

~~$\Rightarrow c_n \in J$~~

$x^2 + y^2 = 0 \Rightarrow x^2 = 0,\ y^2 = 0.$

$a_n^2 + b_n^2 \in J \Rightarrow a_n^2 \in J$

$1 = a_n^2 +$

<u>Claim</u> If $a_n = 0$ on an essential set, then $(a_n) \in J$

If $a_n \neq 0$ on an essential set,

Suppose $a_n \geq 1$ for all n. Is it clear than $(a_n) \notin J$?

$\bar{a} \cdot \bar{b} = \bar{1} \Rightarrow \bar{a} \neq 0.$

$\hat{J} = \{(a_n) : \forall \epsilon > 0 \quad \{n : |a_n| < \epsilon\}$ is essential$\}$

<u>claim</u> $J \subset \hat{J}$

<u>Pf</u> Suppose $(a_n) \in J$, and $\epsilon > 0$ is such that $\{n : |a_n| \geq \epsilon\}$ is essential.

Let $b_n = \begin{cases} \frac{1}{a_n} & |a_n| \gtrsim 0 \\ 0 & \text{otherwise.} \end{cases}$

Then $a_n \cdot b_n = 1$ on an essential set,

so $\overline{a_n}\, \overline{b_n} \neq 0$, so $\overline{a_n} \neq 0$ so $a_n \notin J \Rightarrow\Leftarrow$.

Now by the maximallity of J, $J = \hat{J}$.