Dummit and Foote, Chapters 1-12

July-29-10
7:23 PM

Part I - Group Theory

Chapter 1 - Introduction to Groups

1. Basic Axioms and Examples: Definition of a group, the sillies, Abelian groups, orders of elements, multiplication table. 20 minutes.
2. Dihedral Groups: Definition, generators and relations. 40 minutes.
3. Symmetric Groups: Definition, order, cycles and cycle decomposition, minor algorithms. 25 minutes.
4. Matrix Groups: Almost nothing. 5 minutes.
5. The Quaternion Group: Just the definition. 10 minutes.
6. Homomorphisms and Isomorphisms: Very basics. 30 minutes. (Show the game of 15? Another 15 minutes).
7. Group Actions: Very basics. 15 minutes.

Landmark: Analysis of the Rubik's Cube.

The game of 15? Later?

Chapter 2 - Subgroups

1. Definition and Examples: Definition, the subgroup criterion. 15 minutes.
2. Centralizers and Normalizers, Stabilizers and Kernels: Centralizers are subgroups, the center, normalizers, stabilizers, kernels. 30 minutes but perhaps should be skipped.
3. Cyclic Groups and Cyclic Subgroups: Not so trivial - cyclic subgroups of same order are isomorphic, orders of elements, the number of generators of a cyclic group, subgroups are cyclic, determination of the subgroups. 1 hour.
4. Subgroups Generated by Subsets of a Group: Definition as an intersection and as a closure. 30 minutes.
5. The Lattice of Subgroups of a Group: Mostly some pretty examples. 20 minutes or skip.

Chapter 3 - Quotient Groups and Homomorphisms

1. Definitions and Examples: basic properties of homomorphisms, cosets, cosets partition the group, conjugation, normal subgroups, quotients, normal iff it is a kernel. 60 minutes.
2. More on Cosets and Lagrange's Theorem: Lagrange's theorem, the index G:H, easy corollaries, statement of Cauchy's theorem and Sylow's theorem, the size of a product of subgroups, when is the product of subgroups a subgroup?. 45 minutes.
3. The Isomorphism Theorems: First, second ("diamond"), third, fourth ("lattice"). 90 minutes.
4. Composition Series and the Hölder Program: An Abelian group whose order is divisible by p has an element of order p, simple groups, simple group, composition series, Jordan-Hölder (no proof), stories about classification, solvable groups, solvability of subgroups and quotient groups. 60 minutes.
5. Transpositions and the Alternating Group: The sign of a permutation, the alternating group, cycles and signs. 45 minutes.

do earlier?

Chapter 4 - Group Actions

1. Group Actions and Permutation Representations: 30 minutes.
2. Groups Acting on Themselves by Left Multiplication - Cayley's Theorem: Every group has a representation in a symmetric group, a subgroup of the highest possible index is normal. 30 minutes.
3. Groups Acting on Themselves by Conjugation - the Class Equation: The class equation, groups of order a prime power have a center, analysis of groups of order p^2, conjugacy in S_n, simplicity of A_5, right group actions. 60 minutes.
4. Automorphisms: inner automorphisms, characteristic subgroups, automorphisms of cyclic groups, further examples. 30 minutes.
5. Sylow's Theorem: Proof, groups of order pq, groups of order 30, groups of order 12, groups of order p^2q, groups of order 60. 120 minutes.
6. The Simplicity of A_n: proof. (30 minutes).

Landmark - groups of order 253.

Chapter 5 - Direct and Semidirect Products and Abelian Groups

1. Direct Products: The order of a product, some trivial properties. 10 minutes.
2. The Fundamental Theorem of Finitely Generated Abelian Groups: Proof

postponed, elementary divisors, etc. 30 minutes.

3. Table of Groups of Small Order: Up to 20. 45 minutes.
4. Recognizing Direct Products: Commutators, Abelianization, when's HK=HxK,. 30 minutes.
5. Semidirect Products: Basic definitions, using a homomorphism into Aut(H), complements, groups of order pq, 30, 12, p^3. 60 minutes. p4 completed here.

Chapter 6 - Further Topics in Group Theory

1. p-Groups, Nilpotent Groups, and Solvable Groups: Multiple properties of p-groups, the upper central series, equivalent conditions for nilpotence, Frattini's argument, the lower central series, nilpotence again, the derived series, solvability, solvability of subgroups and quotients, Burnside, Hall, Feit-Thompson, Thompson, a proof of the fundamental theorem of finite Abelian groups: 120 minutes.
2. Applications in Groups of Medium Order: ... 3 hours or skip.
3. A Word on Free Groups: Definition and basic properties, uniqueness, presentations.

Part II - Ring Theory

Chapter 7 - Introduction to Rings

1. Basic Definitions and Examples: definition, zero divisors, units, domains, finite domains are fields, subrings. 40 minutes.
2. Examples: Polynomial Rings, Matrix Rings, and Group Rings: degrees, polynomial rings over a domain are a domain, matrix rings and group rings. 40 minutes.
3. Ring Homomorphisms and Quotient Rings: homomorphisms, image, kernel, ideals, quotients, the first isomorphism theorem, the other isomorphism theorems for rings. 60 minutes.
4. Properties of Ideals: basic stuff, maximal ideals and fields, prime ideals and domains. 45 minutes.
5. Rings of Fractions: 30 minutes.
6. The Chinese Remainder Theorem: (For general rings): 30 minutes.

Chapter 8 - Euclidean Domains, Principal Ideal Domains, and Unique Factorization Domains

1. Euclidean Domains: definition, the Euclidean algorithm, ideals are principal, GCDs, GCDs and the Euclidean algorithm, universal side divisors. 50 minutes.
2. Principal Ideal Domains: definition, GCDs, principal is maximal, polynomials over PIDs, Dedekind-Hasse norms. 40 minutes.
3. Unique Factorization Domains (UFDs): reducible and prime elements, reducible and prime in PIDs, UFDs, reducible and prime in UFDs, GCDs in UFDs, a PID is a UFD, the fundamental theorem of arithmetic. Factorization in the Gaussian Integers: factors of n^2+1, Fermat's theorem on sums of squares and irreducibles in the Gaussian integers. 90 + 60 minutes.

Chapter 9 - Polynomial Rings

1. Definitions and Basic Properties: addition, multiplication, degrees, ideals, many variables. 30 minutes.
2. Polynomial Rings over Fields I: such are Euclidean, F[x] is a PID and a UFD: 20 minutes.
3. Polynomial Rings that are Unique Factorization Domains: Gauss' Lemma, irreducibility over the field of fractions, R is UFD iff R[x] is UFD. 60 minutes.
4. Irreducibility Criteria: linear factors and roots, irreducibility and roots of Z and Q, irreducibility over quotients, Eisenstein's criterion. 90 minutes.
5. Polynomials Rings over Fields II: roots and multiplicities, finite multiplicative subgroups are cyclic, the multiplicative group of Z/nZ. 60 minutes.
6. Polynomial in Several Variables over a Field and Gröbner Bases: Noetherian rings, Hilbert's Basis Theorem, monomial orderings, Gröbner bases, general polynomial division, Buchberger's criterion and algorithm, reduced Gröbner bases, elimination. 120 minutes.

Part III - Modules and Vector Spaces

Chapter 10 - Introduction to Module Theory

1. Basic Definitions and Examples: