Non Commutative Gaussian Elimination @ MAT 1100

By Dror Bar-Natan

Amended from a similar notebook by Dror Bar-Natan and Itai Bar-Natan. The original version is at http://www.math.toronto.edu/~drorbn/Misc/SchreierSimsRubik/.

Pensieve Header: Non Commutative Gaussian Elimination @ MAT 1100 - as on handout + a printout of the filling table. See more at Dror Bar-Natan: Academic Pensieve: 2009-07.

Program 0

```mathematica
{purple = P[18, 27, 36, 4, 5, 6, 7, 8, 9, 3, 11, 12, 13, 14, 15,
    16, 17, 45, 2, 20, 21, 22, 23, 24, 25, 26, 44, 1, 29, 30, 31, 32, 33, 34, 35,
    43, 37, 38, 39, 40, 41, 42, 10, 19, 28, 52, 49, 46, 53, 50, 47, 54, 51, 48],
white = P[1, 2, 3, 4, 5, 6, 16, 25, 34, 10, 11, 9, 15, 24, 33, 39, 17, 18, 19,
    20, 8, 14, 23, 32, 38, 26, 27, 28, 29, 7, 13, 22, 31, 37, 35, 36, 12,
    21, 30, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54],
green = P[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20,
    21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 36, 48, 47, 46, 49, 42, 45, 38, 41, 44, 37, 40, 43, 30, 29, 28, 49, 50, 51, 52, 53, 54],
blue = P[3, 6, 9, 2, 5, 8, 1, 4, 7, 54, 53, 52, 10, 11, 12, 13, 14, 15, 19, 20,
    21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37,
    38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 18, 17, 16],
red = P[13, 2, 3, 22, 5, 6, 31, 8, 9, 12, 21, 30, 37, 14, 15, 16, 17, 18, 11,
    20, 29, 40, 23, 24, 25, 26, 27, 10, 19, 28, 43, 32, 33, 34, 35, 36,
    46, 38, 39, 49, 41, 42, 52, 44, 45, 1, 47, 48, 4, 50, 51, 7, 53, 54],
yellow = P[1, 2, 48, 4, 5, 51, 7, 8, 54, 10, 11, 12, 13, 14, 3, 18, 27, 36, 19,
    20, 21, 22, 23, 6, 17, 26, 35, 28, 29, 30, 31, 32, 9, 16, 25, 34, 37,
    38, 15, 40, 41, 24, 43, 44, 33, 46, 47, 39, 49, 50, 42, 52, 53, 45];

{\text{RecursionLimit} = 2^{16};

n = 54;
P /: p_P ** p[a___] := p[[{a}]];
Inv[p_P] := p @@ Ordering[p];
Feed[p @@ Range[n]] := Null;
Feed[p_P] := Module[{i, j},
    For[i = 1, p[[i]] = i, ++i; j = p[[i]];
    If[Head[s[i, j]] == p,
      Feed[Inv[s[i, j]] ** p],
      (*Else*) s[i, j] = p;
    Do[If[Head[s[k, 1]] == p,
      Feed[s[i, j] ** s[k, 1]];
      Feed[s[k, 1] ** s[i, j]]
    ],
    {k, n}, {i, n}]
  ]
};
```
\begin{verbatim}
(Feed[#]; Product[1 + Length[Select[Range[n], Head[s[i, #]] === \&]], {i, n}] \& /@ gs

{4, 16, 159993501696000, 21119142223872000,
  43252003274489856000, 43252003274489856000}

Images[i_] := {i} - Join - Select[Range[n], Head[s[i, #]] === \&];
ListPlot[
  Join @@ Table[(i, #) \& /@ Images[i], {i, n}],
  AspectRatio \rightarrow 1
]
\end{verbatim}

43 252 003 274 489 856 000 / (8 \times 3^8 \times 12 \times 2^12)

\frac{1}{12}

Feed[purple]
Feed[white]
Feed[green]
Feed[blue]
Feed[red]
Feed[yellow]