 *Final final info on Thursday!

November-25-09
7:01 PM

Basic properties: 0. $\det(I) = 1$.

1. $\det(E^1_{ij}A) = -\det(A)$ $[\det E^1_{ij} = -1]$

"Exchanging two rows flips the sign of det"

2. $\det(E^2_{i,c}A) = c\det A$ $[\det E^2_{i,c} = c]$

"multiplying a row by c multiplies det by c" even for $c=0$!

3. $\det(E^3_{i,j,c}A) = \det A$ $[\det E^3_{i,j,c} = 1]$

"adding c times one row to another does not change det"

Proof later....

Thm Using these properties, the determinant of <u>any</u> $n\times n$ matrix A can be computed.

on board

Corollary All that there is to know about determinants can be deduced from 0-3; also if $\det'$ satisfies 0-3, then $\det' = \det$.

Thm If $A = E_1 \cdots E_n$ is a product of elementary matrices, then $\det A = \det(E_1)\cdot\det(E_2)\cdots\det(E_n)$

Claim For square matrices, AB invertible $\Leftrightarrow A \,\&\, B$ are inv.

$\Leftarrow$ $(AB)^{-1} = B^{-1}A^{-1}$

$\Rightarrow$ $B(AB)^{-1}$ is a right inverse for A, & for square matrices, if $AC = I$ then also $CA = I$

Thm A is invertible iff $\det(A) \neq 0$

Thm $\det A \cdot B = \det A \cdot \det B$

Thm $\det A^T = \det A$

Thm Everything that's true for rows is also true for columns.

Skipped extras: 1. Other formulas for det. (row/col expansions, permutations)

2. A det formula for A^{-1} & Kramer's law.

1. It's in all the books.
2. I've never used it in my life.

... recall the formula for dets & sketch the proof of the basic properties:

$$|(a_{11})| := a_{11}$$

$$\left|\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}\right| := \sum_{j=1}^{n} (-1)^{1+j} a_{1j} \left|\widetilde{A}_{1j}\right|$$

Then prove: 1. Multilinear in the rows.

2. Vanishes if two adjacent rows are equal.

3. Switches sign if two adjacent rows are interchanged.

4. Switches sign whenever two rows are interchanged.

5 $E^{?}_{i,c}$ & $E^{?}_{i,j,c}$ behaviour.