240Algebral-091126, Hour 33: determinants Determinants as on November 21, 2006.

November-25-09
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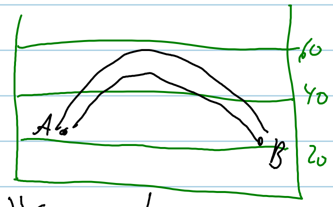
on screen { * "Register of good deeds" 2009, 2006
* on the final exam 2006

on board:

Determinants:

1. Applications: mention 2, prove 1, use none.
2. Formulas: Discuss just one.
3. Basic properties: Our core subject.



"segway happy $\Leftrightarrow$ both wheels are"

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det is a certain specific function, $\det: M_{n\times n}(F)\to F$, which we will properly define later; $\det(A)\equiv|A|$.

1. A invertible $\Longleftrightarrow$ $\det(A)\neq 0$

2. $\det\begin{pmatrix} - r_1 - \\ - r_2 - \\ - r_n - \end{pmatrix} = \mathrm{vol}\left(\begin{array}{c}\text{Parallelopiped generated}\\ \text{by } r_1 \ldots r_n\end{array}\right)$

$$|(a_{11})| := a_{11}$$

$$\left|\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{n1} & \cdots & a_{nn}\end{pmatrix}\right| := \sum_{j=1}^{n} (-1)^{1+j} a_{1j} |\tilde{A}_{1j}|$$

Examples $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

Basic properties: 0. $\det(I)=1$.

1. $\det(E^1_{ij}A) = -\det(A)$ $[\det E^1_{ij} = -1]$

"Exchanging two rows flips the sign of det"

2. $\det(E^2_{i,c}A) = c\det A$ $[\det E^2_{i,c} = c]$

"multiplying a row by c multiplies det by c." even for $c=0$!

3. $\det(E^3_{i,j,c}A) = \det A$ $[\det E^3_{i,j,c} = 1]$

"adding c times one row to another does not change det"

Proof later....

<u>Thm</u> Using these properties, the determinant of <u>any</u> $n \times n$ matrix A can be computed.

<u>Pf</u> Row reduce A keeping track of the affect on $\det A$; for r.r.e.f B, $\det B = 1$ if $B = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$, and $\det B = 0$ if B has a row of 0's.

<u>Examples</u> $\det\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$, $\det\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ done line