240Algebra I-091119, Hour 30: More row reduction

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On board:

Row/col reduction:
* Interchange two rows/cols
* multiply a row/col by $c \neq 0$
* Add c times row/col j to row/col i

* Implemented by $A \mapsto EA, AE$
* Preserves ranks.
* Can reach $\left(\begin{array}{c|c} I_k & 0 \\ \hline 0 & 0 \end{array}\right)$ (rank $= k$)

Suppose you could row reduce A to I. Find A^{-1}.

$$E_4 E_3 E_2 E_1 A = I \implies A^{-1} = E_4 E_3 E_2 E_1$$

* The hard way.
* The easy way: r.r. $(A \mid I)$

Example: compute $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^{-1}$.

How far can you go with row reduction?

(And then with cols? BTW, this is an amusing app. of associativity)

1. The first non-zero entry in each row ("the pivot") is a 1.
2. In the column of a pivot, all else is 0
   [Scan from left to right, to prevent interference]
3. Going down the rows, the pivots are further & further to the right.

"reduced row echelon form" r.r.e.f

Example: $\begin{bmatrix} 1 & 0 & 2 & 9 & 0 & e \\ 0 & 1 & -3 & 7 & 0 & \pi \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Example

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

.... And now with col. ops., can reach $\left(\begin{array}{c|c} I_t & 0 \\ \hline 0 & 0 \end{array}\right)$

<u>Claim</u> The rank of a r.r.e.f matrix is the number of pivots/non-zero rows in it.

<u>Claim</u> If A is invertible, its r.r.e.f. is I