240Algebra I-091020, Hours 17-18

October-16-09
8:55 AM

- APUS
- Final Exam: Wed Dec 16 BN2S 9-12.
- Term exam on Thursday.
- Class notes.

Plan: say all that we can say about lin. trans. without choosing a basis. Then choose bases....

4	3	8
9	5	1
2	7	6

(enough to remember yellow & parities)

Agenda:

- The abstract notion of "isomorphism", the "game of 15" example.
- Any two v.s. of the same dimension are isomorphic.
- Null space / kernel, range / image; they are subspaces.
- Nullity, rank, and rank-nullity.
- Skippable: "the rank is the only invariant of linear transformations".
- TFAE for l.t. between spaces of the same dimension.

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on board

Def V & W are isomorphic if $\exists$ l.t. $T: V \to W$ and $S: W \to V$ s.t. $S \circ T = I_V$ & $T \circ S = I_W$

Thm If V, W are f.d. over F, then $\dim V = \dim W$ iff V is isomorphic to W.

Corollary If $\dim V = n$ over F, V is isomorphic to F^n.

Two "mathematical structures" are "isomorphic" if there's a bijection (1-1 & onto corres.) between their elements which preserves all relevant relations.

Example plastic chess is iso. to ivory chess, but not to checkers.

Example The game of 15.

Pf of Thm & of corollary

Fix a l.t. $T: V \to W$

Def $N(T) = \ker T = \{v : Tv = 0\}$ "null space", "kernel".

$R(T) = \operatorname{im} T = \{Tv : v \in V\}$ "range", "image"

Prop/Def $N(T) \subset V$ is a subspace; $\operatorname{nullity}(T) := \dim N(T)$

$R(T) \subset W$ is a subspace; $\operatorname{rank}(T) := \dim R(T)$

Examples 0, I_V, $D: P_n(\mathbb{R}) \to P_n(\mathbb{R})$

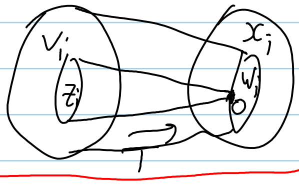
Thm "the dimension theorem". "The rank-nullity Thm"

Thm/ "the dimension theorem", "the rank-nullity Thm"

Given $T: V \to W$, $\underset{m}{\dim V} = \underset{r}{\operatorname{rank}(T)} + \underset{n}{\operatorname{nullity}(T)}$

Pf $(z_i)_1^n$ basis of $N(T)$, extend to $(z_i) \cup (v_i)$ a basis of V,

done line



claim $w_i := T(v_i)$ are lin. indep. in W pf....

claim w_i span $R(T)$ pf...