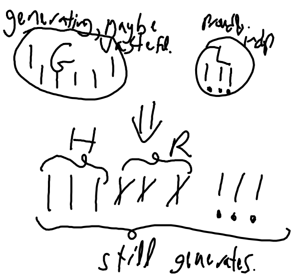
TT discussion: next class?

Corollaries:

on board

1. If V has a finite basis β_1 then every other basis β_2 of V is also finite & $|\beta_1| = |\beta_2|$.
2. "$\dim V$" makes sense.



hour 13

hour 14

3. Assume $\dim V = n$. Then
   a. If G generates V, $|G| \geq n$ & if also $|G| = n$, then G is a basis.
   b. If L is linearly indep in V, the $|L| \leq n$;
      if also $|L| = n$, L is a basis.
      if also $|L| < n$, L can be extended to a basis.
4. If V is finite-dimensional and $W \subset V$ is a subspace, then W is f.d. and $\dim W \leq \dim V$.
   If also $\dim W = \dim V$, then $W = V$.
   If also $\dim W < \dim V$, then any basis of W can be extended to a basis of V.

The Lagrange interpolation formula:

Let x_i be distinct pts in $\mathbb{R}/\mathbb{F}$ $\quad i = 1, \ldots, n+1$

Let y_i be any pts in $\mathbb{R}/\mathbb{F}$.

Q Can you find a polynomial $P \in P_n(\mathbb{R})$ s.t. $P(x_i) = y_i$?

Is it unique?

Who cares? * Scientists.

* Computer drawing programs.

Follow through w/ example.

$p(0) = 5$, $p(1) = 2$, $p(3) = 2$

$P_1 = \frac{(x-1)(x-3)}{3} = \frac{1}{3}(x^2 - 4x + 3)$

$P_2 = \frac{x(x-3)}{-2} =$

$P_3 = \frac{x(x-1)}{6} = \ldots$

$p = x^2 - 4x + 5$

Solution Let $\tilde{P}_i(x) = \prod_{j \neq i} (x - x_j)$

Simplify
$(a-x)(b-x)(c-x) \ldots (z-x)$

Then $p_i(x_j) = \begin{cases} 0 & i \neq j \\ \neq 0 & i = j \end{cases}$

set $p_i(x) = \tilde{p}_i(x) / \tilde{p}_i(x_i) = \ldots$

Then * $p(x) := \sum y_i p_i(x)$ satisfies $p(x_i) = y_i$

* $\beta = \{p_1 \ldots p_{n+1}\}$ is lin. indep.
* $\Rightarrow \beta$ is a basis
* Every $f \in P_n(\mathbb{R})$ can be expressed as a lin. comb. of the p_i in a unique way.
* If $q(x)$ also satisfies $q(x_i) = y_i$, then $q(x) = p(x)$
* Therefore the solution to our problem is unique
* Aside: If $\forall i \; p(x_i) = 0$, then $p = 0$.

(so a non-zero polynomial of degree n may have at most n roots)