Goal: Every V.S. has a "basis". So while we don't <u>have</u> to use coordinates, we can.

on board.

<u>Def</u> $W \subset V$ is a "subspace" if it is a vector space with the operations it inherits from V.

<u>Thm</u> $W \subset V$ is a subspace iff it is "closed under addition and under multiplication by a scalar".

<u>Examples</u> 1. $\{A \in M_{n \times n}(F) : A^t = A\}$

2. $\{A \in M_{n \times n}(F) : \operatorname{tr} A = 0\}$

3. If W_1 & W_2 are subspaces of V, then so is $W_1 \cap W_2$ (what about unions?)

<u>Def</u> u is a l.c. of $u_1 \dots u_n$ if $\exists a_i \in F$ s.t. $u = \sum a_i u_i$

<u>Examples</u> 1. Vitamins as in the handout

2. In $P_3(\mathbb{R})$, $2x^3 - 2x^2 + 12x - 6$ is a l.c. of $x^3 - 2x^2 - 5x - 3$ and $3x^3 - 5x^2 - 4x - 9$

but $3x^3 - 2x^2 + 7x + 8$ isn't.

<u>Thm</u> If $\{u_i\} \subset V$ then $W = \operatorname{span}(u_i) := \{\text{all l.c. of the } u_i\text{'s}\}$ is a subspace of V.

<u>Def</u> $S \subset V$ "generates" or "spans" V. (First requirement from "a basis")

<u>Examples</u> In $V = M_{2\times 2}(\mathbb{R})$ $M_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $M_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $M_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ $M_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ $N_1 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ ... $N_4 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$, Then $M_1 \dots M_4$ & $N_1 \dots N_4$ generate V, but

$M_1 \ldots M_3$ & $N_1 \ldots N_3$ do not.

Aside: If
$S_1 \subset \mathrm{Span}(S_2)$
then
$\mathrm{Span}(S_1) \subset \mathrm{Span}(S_2)$