Hour 1. A calm completion of C:

1. program-proof of “field”
2. Eldar
3. Dimensions

Hour 2. V.S. and subspaces as in textbook.

**Motivation:** For vectors can be added and multiplied by scalars.

**Def** Let $F$ be a field. A v.s. over $F$ is a set $V$, with a special chart over $V$, a binary $+: V \times V \to V$ and a binary $\cdot: F \times V \to V$, s.t.

1. $x + y = y + x$ (VS1: Assoc.)
2. $x + 0 = x$ (VS2: -)
3. $1 \cdot x = x$ (VS3: -)
4. $a(bx) = (ab)x$ (VS4: -)
5. $a(x+y) = (a+b)x$ (VS5: -)

**Examples:**

1. $F^n$
2. $M_{mn}(F)$
3. $F(S,F)$ s a set
4. Polynomials
5. $C/R$, $IR/Q$ “Galois Theory”

**Thm**

1. Cancellation law.
2. $0v$ is unique
3. $v_{-v}$ are unique.
4. $0 \cdot x = 0$
5. $a \cdot 0 = 0$
6. $(-a)x = -(a \cdot x) = a(-x)$
7 \ (-a)x = - (a \cdot x) = a(-x)

Added Oct 28, 2009: I should have also included "CV = 0 \Rightarrow C = 0 \lor V = 0" and "multiplicative cancellation": 
- \( C \neq 0, CV = C'V \Rightarrow V = V' \)
- \( V \neq 0, CV = C'V \Rightarrow C = C' \)