240AlgebraI-090922, Hours 5-6

September-18-09
9:10 AM

Hour 1. A calm completion of $\mathbb{C}$:

1. program-proof of "Field $\overset{?}{\ni} i$ $\supset \mathbb{R}$"
2. Eldar
3. Dimensions
4. Waves.

Hour 2. V.S. and subspaces as in textbook.

<u>Motivation:</u> Forces can be added and multiplied by scalars.

<u>Def</u> Let F be a field. A V.S. over F is a set V, with a special element $0_V \in V$, a binary $+ : V \times V \to V$ and a binary $\cdot : F \times V \to V$, s.t.

VS1. $x+y=y+x$ VS2: Assoc.

VS3. 0 VS4: $-$

VS5: $1\cdot x = x$ VS6 $a(bx)=(ab)x$

VS7 $a(x+y)$ VS8 $(a+b)x$

<u>Examples:</u> 1. F^n

2. $M_{m\times n}(F)$
3. $\mathcal{F}(S,F)$ S a set.
4. Polynomials
5. $\mathbb{C}/\mathbb{R}$ $\mathbb{R}/\mathbb{Q}$ "Galois theory"

<u>Thm</u> 1. Cancellation law.

2. 0_V is unique
3. negatives are unique.

5. $0\cdot x = 0$ 6. $a\cdot 0 = 0$

7 $(-a)x = -(ax) = a(-x)$

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Added Oct 28, 2009: I should have also included "$cv = 0 \Rightarrow c = 0 \vee v = 0$" and "multiplicative cancellation": $c \neq 0,\ cv = cv' \Rightarrow v = v'$

$v \neq 0,\ cv = c'v \Rightarrow c = c'$