

0. Math 240 Algebra I, DROR BAR-NATAN.

1. What we'll do:

The practical side

1. Solve $5x_1 - 2x_2 + x_3 = 9$
 $\begin{pmatrix} \text{how?} \\ \text{when?} \\ \text{one/many?} \end{pmatrix} \begin{matrix} -x_1 + x_2 - x_3 = -2 \\ 2x_1 + 9x_2 - 3x_3 = -4 \end{matrix}$
 this describes the small-scale behaviour of almost everything that has a mathematical description
2. $\hookrightarrow \begin{pmatrix} 5 & -2 & 1 \\ -1 & 1 & -1 \\ 2 & 9 & -3 \end{pmatrix} = A$ "matrices" can add, multiply, take powers: A^{2009}
 This describes the approximate long term behaviour of almost everything - ...

The Theory side.

3. Do all of this in a coordinate-free way!
4. Do all this over "other sets of numbers".
5. Hidden agenda: Learn the basic pure-math processes of abstraction, generalization, definitions, theorems, proofs, notation, logic.

2. Go over the "About" handout.

3 The real numbers: a set \mathbb{R} with two binary ops $+$ & \times and two special elements 0 & 1 s.t.

- done line
- R1 $a+b = b+a$ $ab = ba$
 - R2 ASSOC. R4 negatives & inverses
 - R3 $0, 1$ R5 Distributivity.

Much of algebra, though not all, follows:

Follows: $(a+b)(a-b) = a^2 - b^2$

Doesn't follow: $\forall a \exists x$ s.t. $a = x^2$ or $a = -x^2$

D.r A Final F

Def A Field F

Examples 1. The reals \mathbb{R} .

2. The rationals \mathbb{Q}

3. The complex numbers $\mathbb{C} = \{a + b\sqrt{-1}\}$

4. \mathbb{Z}_2 with $\begin{array}{c|c|c} + & 0 & 1 \\ \hline 0 & 0 & 1 \\ \hline 1 & 1 & 0 \end{array}$ $\begin{array}{c|c|c} + & 0 & 1 \\ \hline 0 & 0 & 0 \\ \hline 1 & 0 & 1 \end{array}$

5. \mathbb{Z}_6 with a funny def. of $+$, \times .