AKT-091203, Hours 33-34: The Existence of Exp

November-30-09
1:11 PM

HW can be picked up on Monday.

Second hour:

* Write the final.
* A discussion of the future.
* Course evaluation.

First hour: Existence of the exponential function.

<u>Thm</u> Assuming a non-obvious result in homological algebra, there exists a power series $e = e(x) \in \mathbb{Q}[[x]]$ w/ $e = 1 + x + \dots$ and $e(x+y) = e(x)e(y)$ (in $\mathbb{Q}[[x,y]]$).

Comments 1. This is a miracle!

2. $\phi \xrightarrow{\Delta} \phi| + |\phi \qquad \Delta e = e^1 e^2$

3. Of course, $e(x) = \sum x^n/n!$

<u>Sketch</u> Assume e_7 solves to degree 7, so

$$e_7(x+y) - e_7(x)e_7(y) = \underbrace{M(x,y)}_{\deg 8} + \text{higher terms.}$$

set $e_8(x) = e_7(x) + \overbrace{\epsilon(x)}^{\deg 8}$ and find that we need to have

$$d^1\epsilon := \epsilon(x) - \epsilon(x+y) + \epsilon(y) = M(x,y).$$

This is hopeless unless M satisfies a relation... Consider

$$\begin{tikzcd}
 & e_7(x+y)e_7(z) \arrow[dr, dash, "M(x,y)"] & \\
e_7(x+y+z) \arrow[ur, dash, "M(x+y,z)"] \arrow[dr, dash, "M(x,y+z)"'] & & e_7(x)e_7(y)e_7(z) \\
 & e_7(x)e_7(y+z) \arrow[ur, dash, "M(y,z)"'] &
\end{tikzcd}$$

(in $\mathbb{Q}[[x,y,z]]$, mod $\deg \geq 9$)

and find that

$$d^2M := M(y,z) - M(x+y,z) + M(x,y+z) - M(x,y) = 0$$

<u>Thm</u> Let $C^n := \mathbb{Q}[[x_1, \dots x_n]]$, $d^n : C^n \to C^{n+1}$ ...

then $H^1 = \mathbb{Q}$ in degree 1, and all other homologies vanish.

Write the exam: Goals:

* Make you review the material.
* Give good grades, but not for free.

Dror's Failures:

1. Failed to cover promised material.
2. Failed to cover even the 4-column in full.
3. Even what was "covered" was not covered well.
4. Failed to connect to new students.
5. Failed to connect between old and new students.
6. Failed to properly push the course web site.
7. Failed to return HW in time.