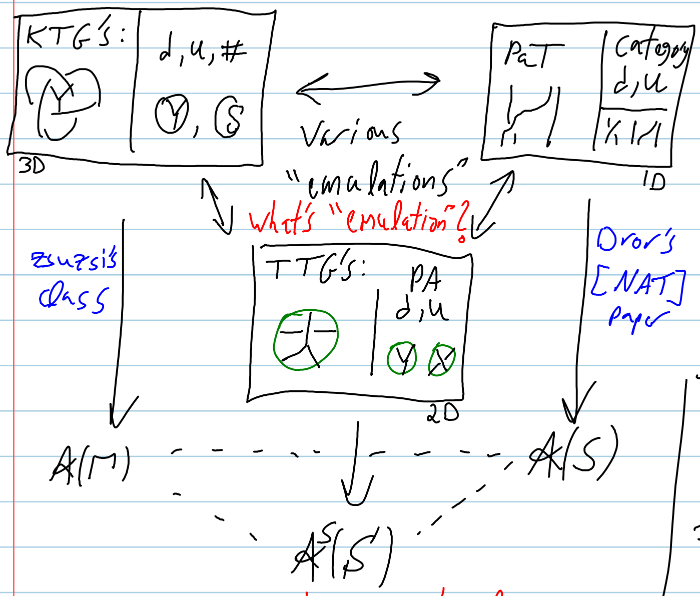
AKT-091124, Hour 29: KTGs, PaT, Shielded Tangles

November-22-09
2:36 PM



Somewhat unstable

glitches & error messages abound

you have been warned

(Why I'm a bit frustrated...)

On board

Peter's class, Thursday or next week

Given $\mathfrak{g} \longrightarrow$ "braided monoidal categories"

The pentagon:

$$\Phi^{123}(1\Delta 1)(\Phi)\cdot\Phi^{234} = (\Delta 1 1)\Phi\cdot(1 1\Delta)(\Phi)$$

The hexagons:

$$(\Delta\otimes 1)(R^{\pm 1}) = \Phi^{123}(R^{\pm1})^{23}(\Phi^{-1})^{132}(R^{\pm1})^{13}\Phi^{312}$$

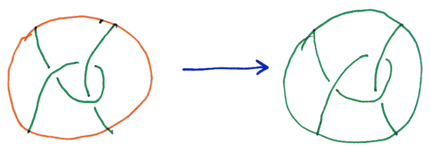
The minors:

$$\Phi^{321}\Phi=1\ ,\quad d_1\Phi=d_2\Phi=d_3\Phi=1\ ,\ \ldots\ldots$$

Aside 1

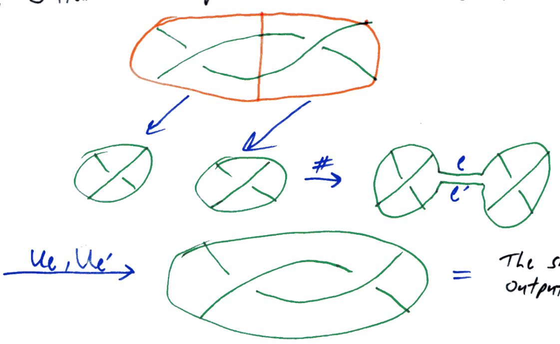
KTG is finitely generated, by [tetrahedron] and [square]:

1. "shield" all tangles:



The shielded generators of tangled graphs.

2. shielded compositions are definable:

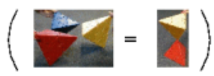


The shielded output.

3. Relations: 1. whatever makes this well-defined.
   2. The Reidemeister moves.
4. So finding a Z is just a matter of finding/guessing Z(tetrahedron) & Z(square), solving a few relations...

Aside 2

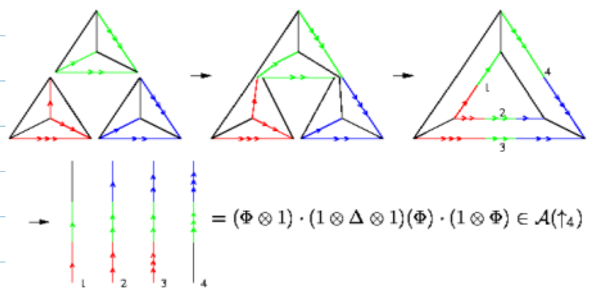
Lest you think it is easy...



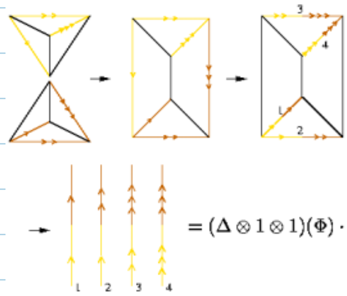
Claim. With $\Phi := Z(\text{tetrahedron})$, the above relation becomes equivalent to the Drinfel'd's pentagon of the theory of quasi Hopf algebras.

Proof.

$:= \Phi \in \mathcal{A}(\uparrow_3)$



$= (\Phi\otimes 1)\cdot(1\otimes\Delta\otimes 1)(\Phi)\cdot(1\otimes\Phi) \in \mathcal{A}(\uparrow_4)$



$= (\Delta\otimes 1\otimes 1)(\Phi)\cdot(1\otimes 1\otimes\Delta)(\Phi) \in \mathcal{A}(\uparrow_4)$