

Def A UFTI $Z: K \rightarrow A$.

Claim Every $W \in A^*$ comes from a V iff

\exists a UFTI $Z: K \rightarrow A$

Filtered v.s. ; the category of filtered v.s.,

graded v.s. ; the category of graded v.s.,

The Functor $gr: F\text{-vect} \rightarrow g\text{-vect}$.

The Functor $fil: g\text{-vect} \rightarrow F\text{-vect}$

$gr \circ fil \cong Id$ but $fil \circ gr \neq Id$

Expansion: $Z: K \rightarrow fil(gr(K))$ s.t.
for $K \in F\text{-vect}$

done line $gr Z: gr K \rightarrow gr K$ is I .

An A -expansion for $K \in F\text{-vect}$, given
 $A \in g\text{-vect}$ and $\pi: A \rightarrow gr K$, is

$Z: K \rightarrow fil A$ s.t. $gr K \xrightarrow{gr Z} A$

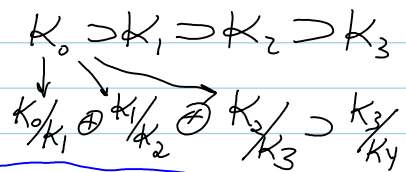
An A -expansion does two things:

1. Proves that π is an iso.
2. Constructs an expansion.

'Adjoint': $K \in F\text{-vect}$
 $A \in g\text{-vect}$
 $\text{Hom}_{g\text{-vect}}(gr K, A) \cong \text{Hom}_{F\text{-vect}}(K, fil A)$
← This map is obvious
→ ?
 $\text{Hom}_g(A, gr K) \cong \text{Hom}_F(fil A, K)$
← obvious.
→ ?

Claim An expansion always exists. [in vort]

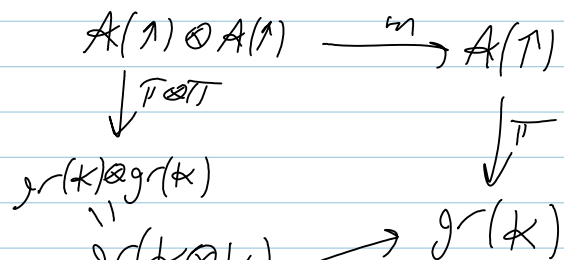
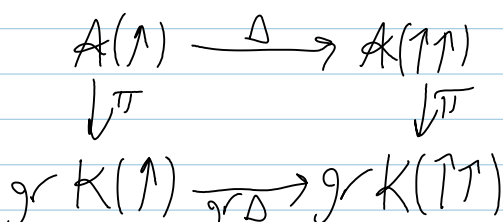
PF



Find K_n^+ in K_n
 K_n^+ in K_{n-1}
so $K_{n-1}/K_n \cong K_n^+$
 $K_0 \cong K_1^+ \oplus K_1 \cong \dots$
 $\cong K_1^+ \oplus K_2^+ \oplus K_3^+ \oplus \dots \oplus K_n$
 $\downarrow \quad \downarrow \quad \downarrow$
 $K_0/K_1 \quad K_1/K_2 \quad K_2/K_3$

If fine, guess maps $m: K \otimes K \rightarrow K$, $\Delta: K(\uparrow) \rightarrow K(\uparrow\uparrow)$
and $\square: K(\uparrow) \rightarrow K(\uparrow) \otimes K(\uparrow)$ s.t. the following diagrams

commute:



$$gr \check{K}(\uparrow) \xrightarrow{gr \Delta} gr \check{K}(\uparrow\uparrow)$$

$$\begin{array}{ccc} gr(K) \otimes gr(K) & & \\ \downarrow & & \\ gr(K \otimes K) & \xrightarrow{gr(m)} & gr(K) \end{array}$$

$$\begin{array}{ccc} A(\uparrow) & \xrightarrow{\square} & A(\uparrow) \otimes A(\uparrow) \\ \downarrow \pi & & \downarrow \\ K(\uparrow) & \xrightarrow{gr \square} & gr(K \otimes K) \end{array}$$