

Pensieve header: Perturbed Alexander-Burau Invariants. DPG code from PerturbedHeisenberg2.nb at pensieve://Classes/21-1350-KnotTheory/. Continued pensieve://Projects/APerturbedAlexanderInvariant/.

```
In[ ]:= Once[<< KnotTheory`];
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at <http://katlas.org/wiki/KnotTheory>.

```
In[ ]:= PD[GST48] = PD[X[1, 15, 2, 14], X[29, 2, 30, 3], X[40, 4, 41, 3],
  X[4, 44, 5, 43], X[5, 26, 6, 27], X[95, 7, 96, 6], X[7, 1, 8, 96], X[8, 14, 9, 13],
  X[28, 9, 29, 10], X[41, 11, 42, 10], X[11, 43, 12, 42], X[12, 27, 13, 28],
  X[15, 31, 16, 30], X[61, 16, 62, 17], X[72, 17, 73, 18], X[83, 18, 84, 19],
  X[34, 20, 35, 19], X[20, 89, 21, 90], X[92, 21, 93, 22], X[22, 79, 23, 80],
  X[23, 68, 24, 69], X[24, 57, 25, 58], X[56, 25, 57, 26], X[31, 63, 32, 62],
  X[32, 74, 33, 73], X[33, 85, 34, 84], X[35, 50, 36, 51], X[81, 37, 82, 36],
  X[70, 38, 71, 37], X[59, 39, 60, 38], X[54, 39, 55, 40], X[55, 45, 56, 44],
  X[45, 59, 46, 58], X[46, 70, 47, 69], X[47, 81, 48, 80], X[91, 49, 92, 48],
  X[49, 91, 50, 90], X[82, 52, 83, 51], X[71, 53, 72, 52], X[60, 54, 61, 53],
  X[74, 63, 75, 64], X[85, 64, 86, 65], X[65, 76, 66, 77], X[66, 87, 67, 88],
  X[94, 67, 95, 68], X[86, 75, 87, 76], X[77, 88, 78, 89], X[93, 78, 94, 79]];
```

```
In[ ]:= ThinPosition[K_] := Module[{todo, done, pd, c},
  todo = List@@PD@K; done = {}; pd = PD[];
  While[todo != {},
    AppendTo[pd, c = RandomChoice@MaximalBy[todo, Length[done ∩ List@@#] &]];
    todo = DeleteCases[todo, c];
    done = done ∪ List@@c];
  pd]
```

```
In[ ]:= RVK::usage =
  "RVK[xs, rots] represents a Rotational Virtual Knot with a list of n Xp/Xm crossings
  xs and a length 2n list of rotation numbers rots. Crossing
  sites are indexed 1 through 2n, and rots[[k]] is the rotation
  between site k-1 and site k. RVK is also a casting operator
  converting to the RVK presentation from other knot presentations.";
```

```

In[ ]:= RVK[pd_PD] := Module[{n, xs, x, rots, front = {1}, k},
  n = Length@pd; rots = Table[0, {2 n}];
  xs = Cases[pd, x_X => { Xp[x[[4]], x[[1]] PositiveQ@x
                        { Xm[x[[2]], x[[1]] True }
  ];
  For[k = 1, k ≤ 2 n, ++k,
    If[FreeQ[front, -k],
      front = Flatten@Replace[front, k → (xs /. {
        Xp[k, L_] | Xm[L_, k] => {L + 1, k + 1, -L},
        Xp[L_, k] | Xm[k, L_] => (++)rots[[L]; {-L, k + 1, L + 1}),
        _Xp | _Xm => {}
      }], {1}],
    Cases[front, k | -k] /. {k, -k} => --rots[[k]];
  ]
];
RVK[xs, rots] ];
RVK[K_] := RVK[PD[K]];

```

```

In[ ]:= 
$$\text{rot}_i[n] := (*\text{rot}_i[n] = *) \begin{cases} \eta_i & n = 0 \\ C_{\$} \text{rot}_i[n-1] // m_{i,\$ \rightarrow i} & n > 0 \\ \bar{C}_{\$} \text{rot}_i[n+1] // m_{i,\$ \rightarrow i} & n < 0 \end{cases}$$


```

```

In[ ]:= Z[K_] := Z[RVK@K];
Z[rvk_RVK] := Module[{g, done, st, c, x, i, j, k},
  g = 1; done = {}; st = Range[2 Length[rvk[[1]]]];
  Do[
    {i, j} = List@@c;
    x = c /. {_Xp => Ri,j, _Xm =>  $\bar{R}_{i,j}$ };
    Do[x = (rot0[rvk[[2], k]] x) // m0,k→k, {k, {i, j}}];
    g *= x;
    Do[
      If[MemberQ[done, k + 1], g = g // mk,k+1→k; st = st /. k + 1 → k];
      If[MemberQ[done, k - 1], g = g // mst[[k-1],k→st[[k-1]]; st = st /. k → st[[k - 1]]],
        {k, {i, j}}];
      done = done ∪ {i, j},
      {c, rvk[[1]]}
    ];
  ];
  Factor@g
];

```

```

In[ ]:= ZF[K_] := Z@ThinPosition@K;

```

```

In[ ]:= $k=1;

```

```
In[ ]:= CCF[ $\mathcal{E}$ _] := ExpandDenominator@ExpandNumerator@Together[ $\mathcal{E}$ ];
```

```
In[ ]:= CF[ $\mathcal{E}$ _List] := CF /@  $\mathcal{E}$ ;
CF[ $\mathcal{E}$ _eSeries] := CF /@  $\mathcal{E}$ ;
CF[ $\mathcal{E}$ _] := Module[
  {vs = Cases[ $\mathcal{E}$ , (p | x |  $\pi$  |  $\xi$ )_,  $\infty$ ]  $\cup$  {p | x |  $\pi$  |  $\xi$ }},
  Total[CoefficientRules[Expand[ $\mathcal{E}$ ], vs] /. (ps_ -> c_)  $\Rightarrow$  CCF[c] (Times @@ vsps)
];
CF[ $\mathcal{E}$ _E] := CF /@  $\mathcal{E}$ ;
CF[ $\gamma$ _I] := CF /@  $\gamma$ ;
CF[Esp[_][ $\mathcal{E}$ S_____]] := CF /@ Esp[ $\mathcal{E}$ S];
```

```
In[ ]:= eSeries /: S1_eSeries  $\equiv$  S2_eSeries :=
  Length[S1] == Length[S2]  $\wedge$  Inner[CF[#1] == CF[#2] &, S1, S2, And];
eSeries[0] := eSeries @@ Table[0, {k, 1}];
eSeries /: S1_eSeries + S2_eSeries :=
  eSeries @@ Table[S1[[k]] + S2[[k]], {k, Min[Length@S1, Length@S2]};
eSeries /: S1_eSeries * S2_eSeries := eSeries @@
  Table[Sum[S1[[j + 1]] * S2[[k - j + 1]], {j, 0, k}], {k, 0, Min[Length@S1, Length@S2] - 1};
eSeries /: c_ * S_eSeries := (c #) & /@ S;
eSeries /:  $\partial_{vs}$  S_eSeries := (s  $\mapsto$   $\partial_{vs}$  s) /@ S;
```

```
In[ ]:= {p*, x*,  $\pi$ *,  $\xi$ *} = { $\pi$ ,  $\xi$ , p, x};
(vs_List)* := (v  $\mapsto$  v*) /@ vs;
(u_i)* := (u*)i;
```

```
In[ ]:= E /: E[ $\omega$ 1_, Q1_, P1_]  $\equiv$  E[ $\omega$ 2_, Q2_, P2_] := CF[ $\omega$ 1 ==  $\omega$ 2]  $\wedge$  CF[Q1 == Q2]  $\wedge$  (P1  $\equiv$  P2);
E /: E[ $\omega$ 1_, Q1_, P1_] E[ $\omega$ 2_, Q2_, P2_] := E[ $\omega$ 1  $\omega$ 2, Q1 + Q2, P1 + P2];
Ed1 -> r1[ $\mathcal{E}$ 1S_____]  $\equiv$  Ed2 -> r2[ $\mathcal{E}$ 2S_____] ^:= (d1 == d2)  $\wedge$  (r1 == r2)  $\wedge$  (E[ $\mathcal{E}$ 1S]  $\equiv$  E[ $\mathcal{E}$ 2S]);
Ed1 -> r1[ $\mathcal{E}$ 1S_____] Ed2 -> r2[ $\mathcal{E}$ 2S_____] ^:= E[(d1  $\cup$  d2) -> (r1  $\cup$  r2)] @@ (E[ $\mathcal{E}$ 1S] E[ $\mathcal{E}$ 2S]);
```

```
In[ ]:= Zip1_{ } = Identity;
Zip1vs_ <  $\mathcal{F}$ _, E[ $\omega$ _, Q_, P_] > := Module[{I, F, G, u, v},
  I = IdentityMatrix@Length@vs;
  F = Table[ $\partial_{u,v}$   $\mathcal{F}$ , {u, vs*}, {v, vs*}];
  G = Table[ $\partial_{u,v}$  Q, {u, vs}, {v, vs}];
  CF /@ <
    vs* . F . Inverse[I - G . F] . vs* / 2,
    E[PowerExpand@Factor[ $\omega$  Det[I - G . F]-1/2], Q - vs . G . vs / 2, P]
  >
];
```

```
In[ ]:= Zip2_{ } = Identity;
Zip2_{vs_} @ <mathcal{F}_-, \mathbb{E}[\omega_-, Q_-, P_-]> := Module[{F, Y, u, v},
  F = Table[\partial_{u,v} \mathcal{F}, {u, vs*}, {v, vs*}];
  Y = Table[\partial_v Q, {v, vs*}];
  CF / @ <mathcal{F}, \mathbb{E}[\omega, Q - Y.vs + Y.F.Y / 2, P /. Thread[vs \to vs + F.Y]]>
]
```

```
In[ ]:= Zip3_{vs_} @ <mathcal{F}_-, \mathbb{E}[\omega_-, Q_-, P_-]> := Module[{Z, u, v, m, j},
  Z[0] = P;
  For[m = 0, m < 2 $k, ++m,
    Z[m + 1] = CF[
      1
      2 (m + 1)
      Sum[\partial_{u,v} \mathcal{F} (\partial_{u,v} Z[m] + Sum[(\partial_u Z[j]) (\partial_v Z[m - j]), {j, 0, m}]), {u, vs}, {v, vs}]]
    ];
  \mathbb{E}[\omega, Q, CF[Sum[Z[m], {m, 0, 2 $k}]] /. Table[v \to 0, {v, vs}]]]
]
```

```
In[ ]:= Zip_{vs_}[\mathcal{F}_-, \mathcal{E}_-] := <mathcal{F}, \mathcal{E}> // Zip1_{vs} // Zip2_{vs} // Zip3_{vs}
```

```
In[ ]:= \mathbb{E}_{d1 \to r1}[\mathcal{E}1s\_ ] // \mathbb{E}_{d2 \to r2}[\mathcal{E}2s\_ ] := Module[{is = r1 \cap d2, lvs},
  lvs = Flatten@Table[{x_{\$i}, p_{\$i}}, {i, is}];
  \mathbb{E}_{(d1 \cup \text{Complement}[d2, is]) \to (r2 \cup \text{Complement}[r1, is])} @@ (Zip_{lvs \cup lvs*}[lvs*.lvs, Times[
    \mathbb{E}[\mathcal{E}1s] /. Table[(v : x | p)_i \to v_{\$i}, {i, is}],
    \mathbb{E}[\mathcal{E}2s] /. Table[(v : \xi | \pi)_i \to v_{\$i}, {i, is}]
  ]])
]
```

```
In[ ]:= \eta_{i_-} := \mathbb{E}_{\{i\} \to \{i\}}[1, 0, eSeries[0]];
m_{i_-, j_- \to k_-} := \mathbb{E}_{\{i, j\} \to \{k\}}[1, -\xi_i \pi_j + (\pi_i + \pi_j) p_k + (\xi_i + \xi_j) x_k, eSeries[0]]
```

```
In[ ]:= AllMonomials[{}, 0] = {1};
AllMonomials[{}, d_Integer] /; d > 0 := {};
AllMonomials[{v_-, vs_--}, d_Integer] :=
  Join @@ Table[v^{d-k} AllMonomials[{vs}, k], {k, 0, d}];
AllMonomials[vs_List, {d_}] := Join @@ Table[AllMonomials[vs, k], {k, 0, d}];
```

```
In[ ]:= Basis[js_List, m_] := Flatten@Outer[Times,
  AllMonomials[Table[p_j, {j, js}], m], AllMonomials[Table[x_j, {j, js}], m]];
Basis[js_List, {m_}] := Flatten@Table[Basis[js, k], {k, 0, m}]
```

```
In[ ]:= GenericCombination[bas_, c_] := bas.Table[c_j, {j, Length@bas}];
GenericCombination[bas_, c_k_] := bas.Table[c_{k,j}, {j, Length@bas}];
```

```
In[ ]:= R0_{i,j}_ := E_{{}->{i,j}}[T^{1/2}, (T - 1) (p_i - p_j) x_j, eSeries @@ Table[0, {k + 1}];
Rp_{i,j}_ := E_{{}->{i,j}}[1, 0,
  eSeries @@ Prepend[0] @ Table[GenericCombination[Basis[{i, j}, {k + 1}], c_k], {k, $k}]];
R_{i,j}_ := Module[{i1, j1}, R0_{i,j} Rp_{i1,j1} // m_{i,i1->i} // m_{j,j1->j}];
```

```
In[ ]:= R0_{i,j}_ := E_{{}->{i,j}}[T^{-1/2}, (T^{-1} - 1) (p_i - p_j) x_j, eSeries @@ Table[0, {k + 1}];
R_{i,j}_ := E_{{}->{i,j}}[1, 0,
  eSeries @@ Prepend[0] @ Table[GenericCombination[Basis[{i, j}, {k + 1}], d_k], {k, $k}]];
R_{i,j}_ := Module[{i1, j1}, R0_{i,j} Rp_{i1,j1} // m_{i,i1->i} // m_{j,j1->j}];
```

```
In[ ]:= C_{i}_ := E_{{}->{i}}[T^{1/2}, 0,
  eSeries @@ Prepend[0] @ Table[GenericCombination[Basis[{i}, {k + 1}], e_k], {k, $k}]];
C_{i}_ := E_{{}->{i}}[T^{-1/2}, 0,
  eSeries @@ Prepend[0] @ Table[GenericCombination[Basis[{i}, {k + 1}], f_k], {k, $k}]];
```

$$\text{In[*]:= CF / @ \{R_{1,2}, \bar{R}_{1,2}, C_1, \bar{C}_1\}$$

$$\text{Out[*]:= } \left\{ \mathbb{E}_{\{\} \rightarrow \{1,2\}} \left[\sqrt{T}, (-1+T) p_1 x_2 + (1-T) p_2 x_2, \in \text{Series} \left[\theta, \right. \right. \right. \\ c_{1,1} + T p_2 x_1 c_{1,4} + p_1 x_1 (c_{1,2} + c_{1,4} - T c_{1,4}) + T p_2 x_2 c_{1,5} + p_1 x_2 (c_{1,3} + c_{1,5} - T c_{1,5}) + T^2 p_2^2 x_1^2 c_{1,12} + \\ p_1 p_2 x_1^2 (T c_{1,9} + 2 T c_{1,12} - 2 T^2 c_{1,12}) + p_1^2 x_1^2 (c_{1,6} + c_{1,9} - T c_{1,9} + c_{1,12} - 2 T c_{1,12} + T^2 c_{1,12}) + \\ T^2 p_2^2 x_1 x_2 c_{1,13} + p_1 p_2 x_1 x_2 (T c_{1,10} + 2 T c_{1,13} - 2 T^2 c_{1,13}) + \\ p_1^2 x_1 x_2 (c_{1,7} + c_{1,10} - T c_{1,10} + c_{1,13} - 2 T c_{1,13} + T^2 c_{1,13}) + T^2 p_2^2 x_2^2 c_{1,14} + \\ \left. \left. \left. p_1 p_2 x_2^2 (T c_{1,11} + 2 T c_{1,14} - 2 T^2 c_{1,14}) + p_1^2 x_2^2 (c_{1,8} + c_{1,11} - T c_{1,11} + c_{1,14} - 2 T c_{1,14} + T^2 c_{1,14}) \right] \right] \right\},$$

$$\mathbb{E}_{\{\} \rightarrow \{1,2\}} \left[\frac{1}{\sqrt{T}}, \frac{(1-T) p_1 x_2}{T} + \frac{(-1+T) p_2 x_2}{T}, \in \text{Series} \left[\theta, \right. \right. \\ d_{1,1} + \frac{p_2 x_1 d_{1,4}}{T} + \frac{p_1 x_1 (T d_{1,2} - d_{1,4} + T d_{1,4})}{T} + \frac{p_2 x_2 d_{1,5}}{T} + \frac{p_1 x_2 (T d_{1,3} - d_{1,5} + T d_{1,5})}{T} + \frac{p_2^2 x_1^2 d_{1,12}}{T^2} + \\ \frac{p_1 p_2 x_1^2 (T d_{1,9} - 2 d_{1,12} + 2 T d_{1,12})}{T^2} + \frac{p_1^2 x_1^2 (T^2 d_{1,6} - T d_{1,9} + T^2 d_{1,9} + d_{1,12} - 2 T d_{1,12} + T^2 d_{1,12})}{T^2} + \\ \frac{p_2^2 x_1 x_2 d_{1,13}}{T^2} + \frac{p_1 p_2 x_1 x_2 (T d_{1,10} - 2 d_{1,13} + 2 T d_{1,13})}{T^2} + \\ \frac{p_1^2 x_1 x_2 (T^2 d_{1,7} - T d_{1,10} + T^2 d_{1,10} + d_{1,13} - 2 T d_{1,13} + T^2 d_{1,13})}{T^2} + \\ \frac{p_2^2 x_2^2 d_{1,14}}{T^2} + \frac{p_1 p_2 x_2^2 (T d_{1,11} - 2 d_{1,14} + 2 T d_{1,14})}{T^2} + \\ \left. \left. \left. \frac{p_1^2 x_2^2 (T^2 d_{1,8} - T d_{1,11} + T^2 d_{1,11} + d_{1,14} - 2 T d_{1,14} + T^2 d_{1,14})}{T^2} \right] \right] \right\},$$

$$\mathbb{E}_{\{\} \rightarrow \{1\}} \left[\sqrt{T}, \theta, \in \text{Series} \left[\theta, e_{1,1} + p_1 x_1 e_{1,2} + p_1^2 x_1^2 e_{1,3} \right] \right],$$

$$\mathbb{E}_{\{\} \rightarrow \{1\}} \left[\frac{1}{\sqrt{T}}, \theta, \in \text{Series} \left[\theta, f_{1,1} + p_1 x_1 f_{1,2} + p_1^2 x_1^2 f_{1,3} \right] \right\}$$

In[*]:= **RMoves** := {

$$(R_{1,2} R_{4,3} R_{5,6} // m_{1,4 \rightarrow 1} // m_{2,5 \rightarrow 2} // m_{3,6 \rightarrow 3}) \equiv (R_{2,3} R_{4,5} R_{1,6} // m_{1,4 \rightarrow 1} // m_{2,5 \rightarrow 2} // m_{3,6 \rightarrow 3}),$$

$$(R_{1,2} \bar{R}_{3,4} // m_{1,3 \rightarrow 1} // m_{2,4 \rightarrow 2}) \equiv (\eta_1 \eta_2),$$

$$(C_1 \bar{C}_2 // m_{1,2 \rightarrow 1}) \equiv \eta_1,$$

$$(R_{1,4} \bar{R}_{5,2} \bar{C}_3 // m_{2,4 \rightarrow 2} // m_{1,3 \rightarrow 1} // m_{1,5 \rightarrow 1}) \equiv \bar{C}_1 \eta_2,$$

$$(C_3 R_{1,2} // m_{2,3 \rightarrow 2} // m_{2,1 \rightarrow 1}) \equiv (\bar{C}_3 R_{1,2} // m_{1,3 \rightarrow 1} // m_{1,2 \rightarrow 1}),$$

$$(\bar{C}_2 R_{1,3} // m_{1,2 \rightarrow 1} // m_{1,3 \rightarrow 1}) \equiv \eta_1, \quad (\bar{C}_2 \bar{R}_{3,1} // m_{1,2 \rightarrow 1} // m_{1,3 \rightarrow 1}) \equiv \eta_1,$$

$$(C_2 \bar{R}_{1,3} // m_{1,2 \rightarrow 1} // m_{1,3 \rightarrow 1}) \equiv \eta_1, \quad (C_2 R_{3,1} // m_{1,2 \rightarrow 1} // m_{1,3 \rightarrow 1}) \equiv \eta_1$$

}

In[*]:= Short [RMoves, 10]

Out[*]//Short=

$$\begin{aligned} & \{ 3 c_{1,1} + T p_2 x_1 c_{1,4} + \ll 37 \gg + \\ & \quad p_1^2 x_3^2 (T^2 c_{1,6} - 2 T^3 c_{1,6} + T^4 c_{1,6} + T c_{1,7} - 2 T^2 c_{1,7} + T^3 c_{1,7} + 2 c_{1,8} - 4 T c_{1,8} + 3 T^2 c_{1,8} + T^2 c_{1,9} - \\ & \quad 3 T^3 c_{1,9} + 3 T^4 c_{1,9} - T^5 c_{1,9} + T c_{1,10} - 3 T^2 c_{1,10} + 3 T^3 c_{1,10} - T^4 c_{1,10} + 2 c_{1,11} - 5 T c_{1,11} + \\ & \quad 5 T^2 c_{1,11} - 2 T^3 c_{1,11} + T^2 c_{1,12} - 4 T^3 c_{1,12} + 6 T^4 c_{1,12} - 4 T^5 c_{1,12} + T^6 c_{1,12} + T c_{1,13} - 4 T^2 c_{1,13} + \\ & \quad 6 T^3 c_{1,13} - 4 T^4 c_{1,13} + T^5 c_{1,13} + 2 c_{1,14} - 6 T c_{1,14} + 8 T^2 c_{1,14} - 6 T^3 c_{1,14} + 2 T^4 c_{1,14}) + \\ & \quad p_2^2 x_3^2 (T^2 c_{1,8} + T^2 c_{1,11} - T^3 c_{1,11} + T^4 c_{1,12} - 2 T^5 c_{1,12} + T^6 c_{1,12} + T^3 c_{1,13} - \\ & \quad 2 T^4 c_{1,13} + T^5 c_{1,13} + 2 T^2 c_{1,14} - 4 T^3 c_{1,14} + 2 T^4 c_{1,14}) = \\ & 3 c_{1,1} + T^2 p_3 x_1 c_{1,4} + p_1 x_1 (2 c_{1,2} + 2 c_{1,4} - 2 T c_{1,4}) + \ll 40 \gg + \\ & \quad p_1 p_2 x_3^2 (T c_{1,11} - T^2 c_{1,11} + 2 T c_{1,14} - 4 T^2 c_{1,14} + 2 T^3 c_{1,14}) + \\ & \quad p_2 p_3 x_3^2 (T^3 c_{1,11} + 4 T^3 c_{1,14} - 4 T^4 c_{1,14}) + \\ & \quad p_2^2 x_3^2 (T^2 c_{1,8} + T^2 c_{1,11} - T^3 c_{1,11} + 2 T^2 c_{1,14} - 4 T^3 c_{1,14} + 2 T^4 c_{1,14}) , \ll 7 \gg , \ll 1 \gg \} \end{aligned}$$

Solving for R, C, \$k = 1

In[*]:= \$k = 1;

{R_{1,2}, C₁}

unknowns = Cases [{R_{1,2}, R̄_{1,2}, C₁, C̄₁}, (c | d | e | f)_{\$k,_, ∞}] // Union

$$\begin{aligned} \text{Out[*]} = & \{ \mathbb{E}_{\{1\} \rightarrow \{1,2\}} [\sqrt{T}, (-1 + T) p_1 x_2 + (1 - T) p_2 x_2, \in \text{Series} [\theta, \\ & \quad c_{1,1} + T p_2 x_1 c_{1,4} + p_1 x_1 (c_{1,2} + c_{1,4} - T c_{1,4}) + T p_2 x_2 c_{1,5} + p_1 x_2 (c_{1,3} + c_{1,5} - T c_{1,5}) + T^2 p_2^2 x_1^2 c_{1,12} + \\ & \quad p_1 p_2 x_1^2 (T c_{1,9} + 2 T c_{1,12} - 2 T^2 c_{1,12}) + p_1^2 x_1^2 (c_{1,6} + c_{1,9} - T c_{1,9} + c_{1,12} - 2 T c_{1,12} + T^2 c_{1,12}) + \\ & \quad T^2 p_2^2 x_1 x_2 c_{1,13} + p_1 p_2 x_1 x_2 (T c_{1,10} + 2 T c_{1,13} - 2 T^2 c_{1,13}) + \\ & \quad p_1^2 x_1 x_2 (c_{1,7} + c_{1,10} - T c_{1,10} + c_{1,13} - 2 T c_{1,13} + T^2 c_{1,13}) + T^2 p_2^2 x_2^2 c_{1,14} + \\ & \quad p_1 p_2 x_2^2 (T c_{1,11} + 2 T c_{1,14} - 2 T^2 c_{1,14}) + p_1^2 x_2^2 (c_{1,8} + c_{1,11} - T c_{1,11} + c_{1,14} - 2 T c_{1,14} + T^2 c_{1,14})]] , \\ & \mathbb{E}_{\{1\} \rightarrow \{1\}} [\sqrt{T}, \theta, \in \text{Series} [\theta, e_{1,1} + p_1 x_1 e_{1,2} + p_1^2 x_1^2 e_{1,3}]] \} \end{aligned}$$

Out[*]= {c_{1,1}, c_{1,2}, c_{1,3}, c_{1,4}, c_{1,5}, c_{1,6}, c_{1,7}, c_{1,8}, c_{1,9}, c_{1,10}, c_{1,11}, c_{1,12}, c_{1,13}, c_{1,14}, d_{1,1}, d_{1,2}, d_{1,3}, d_{1,4}, d_{1,5}, d_{1,6}, d_{1,7}, d_{1,8}, d_{1,9}, d_{1,10}, d_{1,11}, d_{1,12}, d_{1,13}, d_{1,14}, e_{1,1}, e_{1,2}, e_{1,3}, f_{1,1}, f_{1,2}, f_{1,3}}

In[]:= Short [errors = CCF /@ Cases [RMoves, a_ == b_ => a - b], 25]

Out[]//Short=

$$\left\{ \begin{aligned} & T p_1 x_3 c_{1,2} - T^2 p_1 x_3 c_{1,2} + p_1 x_3 c_{1,3} - T p_1 x_3 c_{1,3} - T p_2 x_1 c_{1,4} + T^2 p_2 x_1 c_{1,4} + T p_3 x_1 c_{1,4} - T^2 p_3 x_1 c_{1,4} + \\ & T p_1 x_2 c_{1,4} - T^2 p_1 x_2 c_{1,4} - T p_2 x_2 c_{1,4} + 2 T^2 p_2 x_2 c_{1,4} - T^3 p_2 x_2 c_{1,4} - T^2 p_3 x_2 c_{1,4} + T^3 p_3 x_2 c_{1,4} + \\ & T p_1 x_3 c_{1,4} - 2 T^2 p_1 x_3 c_{1,4} + T^3 p_1 x_3 c_{1,4} + T^2 p_2 x_3 c_{1,4} - T^3 p_2 x_3 c_{1,4} + p_1 x_3 c_{1,5} - T p_1 x_3 c_{1,5} - \\ & 2 p_1^2 x_1 x_2 c_{1,6} + 2 T p_1^2 x_1 x_2 c_{1,6} + 2 T p_1 p_2 x_2^2 c_{1,6} - 2 T^2 p_1 p_2 x_2^2 c_{1,6} + 2 T p_1^2 x_1 x_3 c_{1,6} - 2 T^2 p_1^2 x_1 x_3 c_{1,6} + \\ & T^2 p_1^2 x_3^2 c_{1,6} - 2 T^3 p_1^2 x_3^2 c_{1,6} + T^4 p_1^2 x_3^2 c_{1,6} + 2 T p_1 p_2 x_2 x_3 c_{1,7} - 2 T^2 p_1 p_2 x_2 x_3 c_{1,7} + T p_1^2 x_3^2 c_{1,7} - \\ & 2 T^2 p_1^2 x_3^2 c_{1,7} + T^3 p_1^2 x_3^2 c_{1,7} + 2 p_1^2 x_2 x_3 c_{1,8} - 2 T p_1^2 x_2 x_3 c_{1,8} + p_1^2 x_3^2 c_{1,8} - 4 T p_1^2 x_3^2 c_{1,8} + \\ & 3 T^2 p_1^2 x_3^2 c_{1,8} + 2 T p_1 p_2 x_3^2 c_{1,8} - 2 T^2 p_1 p_2 x_3^2 c_{1,8} - T p_1 p_2 x_1^2 c_{1,9} + T^2 p_1 p_2 x_1^2 c_{1,9} + \langle\langle 212 \rangle\rangle + \\ & 4 T^4 p_1 p_2 x_2 x_3 c_{1,13} - T^2 p_2^2 x_2 x_3 c_{1,13} + 4 T^3 p_2^2 x_2 x_3 c_{1,13} - 4 T^4 p_2^2 x_2 x_3 c_{1,13} + T^5 p_2^2 x_2 x_3 c_{1,13} + \\ & 2 T^3 p_1 p_3 x_2 x_3 c_{1,13} - 2 T^4 p_1 p_3 x_2 x_3 c_{1,13} - 2 T^3 p_2 p_3 x_2 x_3 c_{1,13} + 4 T^4 p_2 p_3 x_2 x_3 c_{1,13} - \\ & 2 T^5 p_2 p_3 x_2 x_3 c_{1,13} - T^4 p_3^2 x_2 x_3 c_{1,13} + T^5 p_3^2 x_2 x_3 c_{1,13} + T p_1^2 x_3^2 c_{1,13} - 4 T^2 p_1^2 x_3^2 c_{1,13} + \\ & 6 T^3 p_1^2 x_3^2 c_{1,13} - 4 T^4 p_1^2 x_3^2 c_{1,13} + T^5 p_1^2 x_3^2 c_{1,13} + 2 T^2 p_1 p_2 x_3^2 c_{1,13} - 6 T^3 p_1 p_2 x_3^2 c_{1,13} + \\ & 6 T^4 p_1 p_2 x_3^2 c_{1,13} - 2 T^5 p_1 p_2 x_3^2 c_{1,13} + T^3 p_2^2 x_3^2 c_{1,13} - 2 T^4 p_2^2 x_3^2 c_{1,13} + T^5 p_2^2 x_3^2 c_{1,13} + 2 p_1^2 x_2 x_3 c_{1,14} - \\ & 6 T p_1^2 x_2 x_3 c_{1,14} + 6 T^2 p_1^2 x_2 x_3 c_{1,14} - 2 T^3 p_1^2 x_2 x_3 c_{1,14} + 4 T p_1 p_2 x_2 x_3 c_{1,14} - 8 T^2 p_1 p_2 x_2 x_3 c_{1,14} + \\ & 4 T^3 p_1 p_2 x_2 x_3 c_{1,14} + 2 T^2 p_2^2 x_2 x_3 c_{1,14} - 2 T^3 p_2^2 x_2 x_3 c_{1,14} + p_1^2 x_3^2 c_{1,14} - 4 T p_1^2 x_3^2 c_{1,14} + 7 T^2 p_1^2 x_3^2 c_{1,14} - \\ & 6 T^3 p_1^2 x_3^2 c_{1,14} + 2 T^4 p_1^2 x_3^2 c_{1,14} + 2 T p_1 p_2 x_3^2 c_{1,14} - 6 T^2 p_1 p_2 x_3^2 c_{1,14} + 6 T^3 p_1 p_2 x_3^2 c_{1,14} - \\ & 2 T^4 p_1 p_2 x_3^2 c_{1,14} + 2 T^3 p_1 p_3 x_3^2 c_{1,14} - 2 T^4 p_1 p_3 x_3^2 c_{1,14} - 2 T^3 p_2 p_3 x_3^2 c_{1,14} + 2 T^4 p_2 p_3 x_3^2 c_{1,14}, \\ & \frac{T^2 c_{\langle\langle 1 \rangle\rangle} + \langle\langle 154 \rangle\rangle + \langle\langle 1 \rangle\rangle}{T^2}, \langle\langle 1 \rangle\rangle, \langle\langle 3 \rangle\rangle, d_{\langle\langle 1 \rangle\rangle} + \langle\langle 54 \rangle\rangle + \langle\langle 1 \rangle\rangle, \\ & \left. \frac{\langle\langle 1 \rangle\rangle}{T^2}, \frac{T^2 c_{1,1} + \langle\langle 54 \rangle\rangle + p_1^2 x_1^2 e_{1,3}}{T^2} \right\} \end{aligned} \right.$$

In[]:= Short [eqns = Thread[0 == Union @@ (CoefficientRules[#, {x1, x2, x3, p1, p2, p3}][[; ; , 2] & /@ errors)], 10]

Out[]//Short=

$$\left\{ \begin{aligned} & \theta = T c_{1,4} - T^2 c_{1,4}, \theta = -T c_{1,4} + T^2 c_{1,4}, \theta = T^2 c_{1,4} - T^3 c_{1,4}, \\ & \theta = -T c_{1,4} + 2 T^2 c_{1,4} - T^3 c_{1,4}, \theta = -T^2 c_{1,4} + T^3 c_{1,4}, \langle\langle 74 \rangle\rangle, \\ & \theta = c_{1,6} - T^2 c_{1,6} + \frac{c_{1,7}}{T} - T c_{1,7} - c_{1,8} + \frac{c_{1,8}}{T^2} + c_{1,9} - T^2 c_{1,9} + \frac{c_{1,10}}{T} - T c_{1,10} - \\ & c_{1,11} + \frac{c_{1,11}}{T^2} + c_{1,12} - T^2 c_{1,12} + \frac{c_{1,13}}{T} - T c_{1,13} - c_{1,14} + \frac{c_{1,14}}{T^2} + \frac{e_{1,3}}{T^2} - T^2 f_{1,3}, \\ & \theta = T^2 c_{1,6} + T c_{1,7} + c_{1,8} + T^2 c_{1,9} + T c_{1,10} + c_{1,11} + T^2 c_{1,12} + T c_{1,13} + c_{1,14} + T^2 f_{1,3}, \\ & \theta = d_{1,6} + T d_{1,7} + T^2 d_{1,8} + d_{1,9} + T d_{1,10} + T^2 d_{1,11} + d_{1,12} + T d_{1,13} + T^2 d_{1,14} + T^2 f_{1,3}, \\ & \theta = c_{1,6} - 2 T c_{1,6} + T^2 c_{1,6} - c_{1,7} + T c_{1,7} + c_{1,8} + T^2 d_{1,8} - T d_{1,11} + T^2 d_{1,11} + d_{1,14} - 2 T d_{1,14} + \\ & T^2 d_{1,14} + f_{1,3} - 2 T f_{1,3} + T^2 f_{1,3}, \theta = d_{1,1} - T d_{1,3} + d_{1,5} - T d_{1,5} + 2 T^2 d_{1,8} - 2 T d_{1,11} + \\ & 2 T^2 d_{1,11} + 2 d_{1,14} - 4 T d_{1,14} + 2 T^2 d_{1,14} + f_{1,1} + f_{1,2} - T f_{1,2} + 2 f_{1,3} - 4 T f_{1,3} + 2 T^2 f_{1,3} \end{aligned} \right.$$

In[]:= **{sol} = Solve[eqns, unknowns]**

Solve: Equations may not give solutions for all "solve" variables.

$$\text{Out[]} = \left\{ \left\{ \begin{aligned} c_{1,1} &\rightarrow -\frac{c_{1,2}}{2} - \frac{c_{1,5}}{2}, c_{1,3} \rightarrow -T c_{1,2} - c_{1,5}, c_{1,4} \rightarrow \theta, c_{1,6} \rightarrow \theta, c_{1,7} \rightarrow -\frac{1}{2} (1+T) c_{1,10} - c_{1,11}, \\ c_{1,8} &\rightarrow -\frac{1}{2} (T - T^2) c_{1,10} - (1-T) c_{1,11}, c_{1,9} \rightarrow \theta, c_{1,12} \rightarrow \theta, c_{1,13} \rightarrow \theta, c_{1,14} \rightarrow \theta, \\ d_{1,1} &\rightarrow \frac{c_{1,2}}{2} + \frac{c_{1,5}}{2}, d_{1,2} \rightarrow -c_{1,2}, d_{1,3} \rightarrow \frac{c_{1,2}}{T} + c_{1,5}, d_{1,4} \rightarrow \theta, d_{1,5} \rightarrow -c_{1,5}, d_{1,6} \rightarrow \theta, \\ d_{1,7} &\rightarrow -\frac{(1-3T) c_{1,10}}{2T} + \frac{c_{1,11}}{T}, d_{1,8} \rightarrow -\frac{(-1+3T-2T^2) c_{1,10}}{2T^2} - \frac{(1-T) c_{1,11}}{T^2}, d_{1,9} \rightarrow \theta, \\ d_{1,10} &\rightarrow -c_{1,10}, d_{1,11} \rightarrow -\frac{(-1+T) c_{1,10}}{T} - \frac{c_{1,11}}{T}, d_{1,12} \rightarrow \theta, d_{1,13} \rightarrow \theta, d_{1,14} \rightarrow \theta, \\ e_{1,1} &\rightarrow -\frac{c_{1,2}}{2} - \frac{c_{1,5}}{2}, e_{1,2} \rightarrow -c_{1,10}, e_{1,3} \rightarrow \theta, f_{1,1} \rightarrow \frac{c_{1,2}}{2} + \frac{c_{1,5}}{2}, f_{1,2} \rightarrow c_{1,10}, f_{1,3} \rightarrow \theta \end{aligned} \right\} \right\}$$

In[]:= **sol /. (a_ -> b_) :-> (a = b)**

$$\text{Out[]} = \left\{ \begin{aligned} &-\frac{c_{1,2}}{2} - \frac{c_{1,5}}{2}, -T c_{1,2} - c_{1,5}, \theta, \theta, -\frac{1}{2} (1+T) c_{1,10} - c_{1,11}, \\ &-\frac{1}{2} (T - T^2) c_{1,10} - (1-T) c_{1,11}, \theta, \theta, \theta, \theta, \frac{c_{1,2}}{2} + \frac{c_{1,5}}{2}, -c_{1,2}, \frac{c_{1,2}}{T} + c_{1,5}, \theta, \\ &-c_{1,5}, \theta, -\frac{(1-3T) c_{1,10}}{2T} + \frac{c_{1,11}}{T}, -\frac{(-1+3T-2T^2) c_{1,10}}{2T^2} - \frac{(1-T) c_{1,11}}{T^2}, \theta, -c_{1,10}, \\ &-\frac{(-1+T) c_{1,10}}{T} - \frac{c_{1,11}}{T}, \theta, \theta, \theta, -\frac{c_{1,2}}{2} - \frac{c_{1,5}}{2}, -c_{1,10}, \theta, \frac{c_{1,2}}{2} + \frac{c_{1,5}}{2}, c_{1,10}, \theta \end{aligned} \right\}$$

In[]:= **CF /@ {Rp_{1,2}, R̄p_{1,2}, C₁, C̄₁}**

$$\text{Out[]} = \left\{ \begin{aligned} &E_{\{\} \rightarrow \{1,2\}} \left[1, \theta, \right. \\ &\quad \in \text{Series} \left[\theta, p_1 x_1 c_{1,2} + \frac{1}{2} (-c_{1,2} - c_{1,5}) + p_1 x_2 (-T c_{1,2} - c_{1,5}) + p_2 x_2 c_{1,5} + p_1 p_2 x_1 x_2 c_{1,10} + \frac{1}{2} p_1^2 x_1 \right. \\ &\quad \quad \left. x_2 (-c_{1,10} - T c_{1,10} - 2 c_{1,11}) + p_1 p_2 x_2^2 c_{1,11} + \frac{1}{2} p_1^2 x_2^2 (-T c_{1,10} + T^2 c_{1,10} - 2 c_{1,11} + 2 T c_{1,11}) \right] \left. \right], \\ &E_{\{\} \rightarrow \{1,2\}} \left[1, \theta, \in \text{Series} \left[\theta, -p_1 x_1 c_{1,2} - p_2 x_2 c_{1,5} + \frac{1}{2} (c_{1,2} + c_{1,5}) + \frac{p_1 x_2 (c_{1,2} + T c_{1,5})}{T} - \right. \right. \\ &\quad \left. \left. p_1 p_2 x_1 x_2 c_{1,10} + \frac{p_1 p_2 x_2^2 (c_{1,10} - T c_{1,10} - c_{1,11})}{T} + \frac{p_1^2 x_1 x_2 (-c_{1,10} + 3 T c_{1,10} + 2 c_{1,11})}{2 T} + \right. \right. \\ &\quad \quad \left. \left. \frac{p_1^2 x_2^2 (c_{1,10} - 3 T c_{1,10} + 2 T^2 c_{1,10} - 2 c_{1,11} + 2 T c_{1,11})}{2 T^2} \right] \right], \\ &E_{\{\} \rightarrow \{1\}} \left[\sqrt{T}, \theta, \in \text{Series} \left[\theta, \frac{1}{2} (-c_{1,2} - c_{1,5}) - p_1 x_1 c_{1,10} \right] \right], \\ &E_{\{\} \rightarrow \{1\}} \left[\frac{1}{\sqrt{T}}, \theta, \in \text{Series} \left[\theta, \frac{1}{2} (c_{1,2} + c_{1,5}) + p_1 x_1 c_{1,10} \right] \right] \end{aligned} \right\}$$

In[*]:= Factor /@ CF [Last@ZF@Knot [6, 1] - Last@ZF@Knot [9, 46]]

$$\text{Out[*]} = \text{Series} \left[0, \frac{2 (-1 + T)^2 (1 - 4 T + T^2) c_{1,10}}{(-2 + T)^2 (-1 + 2 T)^2} \right]$$

In[*]:= Cases [{Rp_{1,2}, R_{1,2}, C₁, C₁}, (c | d | e | f)_{\$k,_, ∞}] // Union

$$\text{Out[*]} = \{C_{1,2}, C_{1,5}, C_{1,10}, C_{1,11}\}$$

In[*]:= CF /@ ({Rp_{1,2}, R_{1,2}, C₁, C₁ } /. {c_{1,10} → -1/2, c_{1,2} → 0, c_{1,5} → 0})

$$\begin{aligned} \text{Out[*]} = & \left\{ \mathbb{E}_{\{\} \rightarrow \{1,2\}} \left[1, 0, \text{Series} \left[0, \right. \right. \right. \\ & \left. \left. \left. -\frac{1}{2} p_1 p_2 x_1 x_2 + \frac{1}{4} p_1^2 x_1 x_2 (1 + T - 4 c_{1,11}) + p_1 p_2 x_2^2 c_{1,11} + \frac{1}{4} p_1^2 x_2^2 (T - T^2 - 4 c_{1,11} + 4 T c_{1,11}) \right] \right], \right. \\ & \mathbb{E}_{\{\} \rightarrow \{1,2\}} \left[1, 0, \text{Series} \left[0, \frac{1}{2} p_1 p_2 x_1 x_2 + \frac{p_1 p_2 x_2^2 (-1 + T - 2 c_{1,11})}{2 T} + \right. \right. \\ & \left. \left. \frac{p_1^2 x_1 x_2 (1 - 3 T + 4 c_{1,11})}{4 T} + \frac{p_1^2 x_2^2 (-1 + 3 T - 2 T^2 - 4 c_{1,11} + 4 T c_{1,11})}{4 T^2} \right] \right], \\ & \left. \mathbb{E}_{\{\} \rightarrow \{1\}} \left[\sqrt{T}, 0, \text{Series} \left[0, \frac{p_1 x_1}{2} \right] \right], \mathbb{E}_{\{\} \rightarrow \{1\}} \left[\frac{1}{\sqrt{T}}, 0, \text{Series} \left[0, -\frac{1}{2} p_1 x_1 \right] \right] \right\} \end{aligned}$$

In[*]:= CF /@ ({Rp_{1,2}, R_{1,2}, C₁, C₁ } /. {c_{1,10} → -1/2, c_{1,2} → 0, c_{1,5} → 0, c_{1,11} → T/2})

$$\begin{aligned} \text{Out[*]} = & \left\{ \mathbb{E}_{\{\} \rightarrow \{1,2\}} \left[1, 0, \text{Series} \left[0, \frac{1}{4} (1 - T) p_1^2 x_1 x_2 - \frac{1}{2} p_1 p_2 x_1 x_2 + \frac{1}{4} (-T + T^2) p_1^2 x_2^2 + \frac{1}{2} T p_1 p_2 x_2^2 \right] \right], \right. \\ & \mathbb{E}_{\{\} \rightarrow \{1,2\}} \left[1, 0, \text{Series} \left[0, \frac{(1 - T) p_1^2 x_1 x_2}{4 T} + \frac{1}{2} p_1 p_2 x_1 x_2 + \frac{(-1 + T) p_1^2 x_2^2}{4 T^2} - \frac{p_1 p_2 x_2^2}{2 T} \right] \right], \\ & \left. \mathbb{E}_{\{\} \rightarrow \{1\}} \left[\sqrt{T}, 0, \text{Series} \left[0, \frac{p_1 x_1}{2} \right] \right], \mathbb{E}_{\{\} \rightarrow \{1\}} \left[\frac{1}{\sqrt{T}}, 0, \text{Series} \left[0, -\frac{1}{2} p_1 x_1 \right] \right] \right\} \end{aligned}$$

In[*]:= {c_{1,10} = -1/2, c_{1,2} = 0, c_{1,5} = 0, c_{1,11} = T/2};
CF /@ {R_{1,2}, R_{1,2}, C₁, C₁}

$$\begin{aligned} \text{Out[*]} = & \left\{ \mathbb{E}_{\{\} \rightarrow \{1,2\}} \left[\sqrt{T}, (-1 + T) p_1 x_2 + (1 - T) p_2 x_2, \right. \right. \\ & \left. \left. \text{Series} \left[0, \frac{1}{4} (-1 + T) p_1^2 x_1 x_2 - \frac{1}{2} T p_1 p_2 x_1 x_2 + \frac{1}{4} (T - T^2) p_1^2 x_2^2 + \frac{1}{2} T^2 p_1 p_2 x_2^2 \right] \right], \right. \\ & \mathbb{E}_{\{\} \rightarrow \{1,2\}} \left[\frac{1}{\sqrt{T}}, \frac{(1 - T) p_1 x_2}{T} + \frac{(-1 + T) p_2 x_2}{T}, \right. \\ & \left. \left. \text{Series} \left[0, \frac{(-1 + T) p_1^2 x_1 x_2}{4 T} + \frac{p_1 p_2 x_1 x_2}{2 T} + \frac{(1 - T) p_1^2 x_2^2}{4 T^2} - \frac{p_1 p_2 x_2^2}{2 T^2} \right] \right], \right. \\ & \left. \mathbb{E}_{\{\} \rightarrow \{1\}} \left[\sqrt{T}, 0, \text{Series} \left[0, \frac{p_1 x_1}{2} \right] \right], \mathbb{E}_{\{\} \rightarrow \{1\}} \left[\frac{1}{\sqrt{T}}, 0, \text{Series} \left[0, -\frac{1}{2} p_1 x_1 \right] \right] \right\} \end{aligned}$$

$$\text{In[*]} := \text{CF}[\mathbf{R}_{1,2} / \cdot \mathbf{T} \rightarrow \mathbf{T}^{-1}]$$

$$\text{Out[*]} = \mathbb{E}_{\{\} \rightarrow \{1,2\}} \left[\sqrt{\frac{1}{\mathbf{T}}}, \frac{(1-\mathbf{T}) p_1 x_2}{\mathbf{T}} + \frac{(-1+\mathbf{T}) p_2 x_2}{\mathbf{T}}, \right. \\ \left. \in \text{Series} \left[\mathbf{0}, \frac{(1-\mathbf{T}) p_1^2 x_1 x_2}{4 \mathbf{T}} - \frac{p_1 p_2 x_1 x_2}{2 \mathbf{T}} + \frac{(-1+\mathbf{T}) p_1^2 x_2^2}{4 \mathbf{T}^2} + \frac{p_1 p_2 x_2^2}{2 \mathbf{T}^2} \right] \right]$$

$$\text{In[*]} := \text{RMoves}$$

$$\text{Out[*]} = \{\text{True}, \text{True}, \text{True}, \text{True}, \text{True}, \text{True}, \text{True}, \text{True}, \text{True}\}$$

The isomorphism with Γ -calculus

$$\text{In[*]} := \Gamma[\mathbb{E}_{\{\} \rightarrow \text{is}}[\omega, \mathbf{Q}, _]] := \text{CF@}\Gamma[\omega^{-1}, \text{Sum}[p_i x_i, \{\mathbf{i}, \text{is}\}] - \mathbf{Q}] \\ \mathbb{E}[\Gamma[\omega, \mathbf{A}]] := \text{Module}[\{\text{is} = \text{Union@Cases}[\mathbf{A}, p_i \rightarrow \mathbf{i}, \infty]\}, \\ \text{CF@}\mathbb{E}_{\{\} \rightarrow \text{is}}[\omega^{-1}, \text{Sum}[p_i x_i, \{\mathbf{i}, \text{is}\}] - \mathbf{A}, \text{eSeries}[\mathbf{0}]]]$$

$$\text{In[*]} := \{\mathbf{R}_{i,j}, \mathbf{R}_{i,j} // \Gamma, \mathbf{R}_{i,j} // \Gamma // \mathbb{E}\} // \text{Column}$$

$$\mathbb{E}_{\{\} \rightarrow \{i,j\}} \left[\sqrt{\mathbf{T}}, (-1+\mathbf{T}) p_i x_j + (1-\mathbf{T}) p_j x_j, \right. \\ \left. \in \text{Series} \left[\mathbf{0}, \frac{1}{4} (-1+\mathbf{T}) p_i^2 x_i x_j - \frac{1}{2} \mathbf{T} p_i p_j x_i x_j + \frac{1}{4} (\mathbf{T} - \mathbf{T}^2) p_i^2 x_j^2 + \frac{1}{2} \mathbf{T}^2 p_i p_j x_j^2 \right] \right] \\ \text{Out[*]} = \Gamma \left[\frac{1}{\sqrt{\mathbf{T}}}, p_i x_i + (1-\mathbf{T}) p_i x_j + \mathbf{T} p_j x_j \right] \\ \mathbb{E}_{\{\} \rightarrow \{i,j\}} \left[\sqrt{\mathbf{T}}, (-1+\mathbf{T}) p_i x_j + (1-\mathbf{T}) p_j x_j, \in \text{Series}[\mathbf{0}, \mathbf{0}] \right]$$

$$\text{In[*]} := \gamma \mathbf{1} = \Gamma[\omega, \{x_i, x_j, x_r\} \cdot \begin{pmatrix} \alpha & \beta & \theta \\ \gamma & \delta & \epsilon \\ \phi & \psi & \xi \end{pmatrix} \cdot \{p_i, p_j, p_r\}] // \mathbb{E}$$

$$\text{Out[*]} = \mathbb{E}_{\{\} \rightarrow \{i,j,r\}} \left[\frac{1}{\omega}, (1-\alpha) p_i x_i - \beta p_j x_i - \theta p_r x_i - \gamma p_i x_j + \right. \\ \left. (1-\delta) p_j x_j - \epsilon p_r x_j - \phi p_i x_r - \psi p_j x_r + (1-\xi) p_r x_r, \in \text{Series}[\mathbf{0}, \mathbf{0}] \right]$$

$$\text{In[*]} := \gamma \mathbf{1} // \mathbf{m}_{i,j \rightarrow k} // \Gamma$$

$$\gg \frac{1}{\sqrt{1-2\beta+\beta^2} \omega}$$

$$\gg \frac{1}{(-1+\beta) \omega}$$

$$\text{Out[*]} = \Gamma \left[-\omega + \beta \omega, \right. \\ \left. \frac{(-\gamma + \beta \gamma - \alpha \delta) p_k x_k}{-1+\beta} + \frac{(-\epsilon + \beta \epsilon - \delta \theta) p_r x_r}{-1+\beta} + \frac{(-\phi + \beta \phi - \alpha \psi) p_k x_r}{-1+\beta} + \frac{(-\xi + \beta \xi - \theta \psi) p_r x_r}{-1+\beta} \right]$$

$$\text{In[*]:= Simplify}\left[\left(\mathbb{E}@\Gamma\left[\omega, \{x_1, x_2, x_3\} \cdot \begin{pmatrix} \alpha & \beta & \gamma \\ \delta & \epsilon & \eta \\ \lambda & \mu & \nu \end{pmatrix} \cdot \{p_1, p_2, p_3\}\right] \mathbb{E}_{\{\} \rightarrow \{i, j\}}[1, \theta, \text{eSeries}[\theta, p_i x_i]] // m_{1, i \rightarrow 1} // m_{1, 2 \rightarrow 1} // m_{1, j \rightarrow 1} // m_{1, 3 \rightarrow 1}\right) \llbracket 3, 2 \rrbracket /. (x | p) _ \rightarrow \theta\right]$$

$$\text{Out[*]} = \frac{\beta + \gamma \epsilon - \beta \eta}{-1 + \beta + \gamma \epsilon + \eta - \beta \eta}$$

$$\text{In[*]:= Simplify}\left[\left(\mathbb{E}@\Gamma\left[\omega, \{x_1, x_2, x_3\} \cdot \begin{pmatrix} \alpha & \beta & \gamma \\ \delta & \epsilon & \eta \\ \lambda & \mu & \nu \end{pmatrix} \cdot \{p_1, p_2, p_3\}\right] \mathbb{E}_{\{\} \rightarrow \{i, j\}}[1, \theta, \text{eSeries}[\theta, p_i x_j]] // m_{1, i \rightarrow 1} // m_{1, 2 \rightarrow 1} // m_{1, j \rightarrow 1} // m_{1, 3 \rightarrow 1}\right) \llbracket 3, 2 \rrbracket /. (x | p) _ \rightarrow \theta\right]$$

$$\text{Out[*]} = \frac{\gamma}{-1 + \beta + \gamma \epsilon + \eta - \beta \eta}$$

$$\text{In[*]:= Simplify}\left[\left(\mathbb{E}@\Gamma\left[\omega, \{x_1, x_2, x_3\} \cdot \begin{pmatrix} \alpha & \beta & \gamma \\ \delta & \epsilon & \eta \\ \lambda & \mu & \nu \end{pmatrix} \cdot \{p_1, p_2, p_3\}\right] \mathbb{E}_{\{\} \rightarrow \{i, j\}}[1, \theta, \text{eSeries}[\theta, p_j x_i]] // m_{1, i \rightarrow 1} // m_{1, 2 \rightarrow 1} // m_{1, j \rightarrow 1} // m_{1, 3 \rightarrow 1}\right) \llbracket 3, 2 \rrbracket /. (x | p) _ \rightarrow \theta\right]$$

$$\text{Out[*]} = \frac{\epsilon}{-1 + \beta + \gamma \epsilon + \eta - \beta \eta}$$

$$\text{In[*]:= Simplify}\left[\left(\mathbb{E}@\Gamma\left[\omega, \{x_1, x_2, x_3\} \cdot \begin{pmatrix} \alpha & \beta & \gamma \\ \delta & \epsilon & \eta \\ \lambda & \mu & \nu \end{pmatrix} \cdot \{p_1, p_2, p_3\}\right] \mathbb{E}_{\{\} \rightarrow \{i, j\}}[1, \theta, \text{eSeries}[\theta, p_j x_j]] // m_{1, i \rightarrow 1} // m_{1, 2 \rightarrow 1} // m_{1, j \rightarrow 1} // m_{1, 3 \rightarrow 1}\right) \llbracket 3, 2 \rrbracket /. (x | p) _ \rightarrow \theta\right]$$

$$\text{Out[*]} = \frac{\gamma \epsilon + \eta - \beta \eta}{-1 + \beta + \gamma \epsilon + \eta - \beta \eta}$$

$$\text{In[*]:= } C_i$$

$$\text{Out[*]} = \mathbb{E}_{\{\} \rightarrow \{i\}}\left[\sqrt{T}, \theta, \text{eSeries}\left[\theta, \frac{p_i x_i}{2}\right]\right]$$

In[*]:= **CF** [**RP**_{i,j} /. T → T⁻¹]

Out[*]:= $\mathbb{E}_{\{\} \rightarrow \{i,j\}} \left[1, \theta, \in \text{Series} \left[\theta, \frac{1}{4} (-1 + T) p_i^2 x_i x_j + \frac{1}{2} p_i p_j x_i x_j + \frac{1}{4} (T - T^2) p_i^2 x_j^2 - \frac{1}{2} T p_i p_j x_j^2 \right] \right]$

In[*]:= **CF** [**RP**_{i,j}]

Out[*]:= $\mathbb{E}_{\{\} \rightarrow \{i,j\}} \left[1, \theta, \in \text{Series} \left[\theta, \frac{1}{4} (1 - T) p_i^2 x_i x_j - \frac{1}{2} p_i p_j x_i x_j + \frac{1}{4} (-T + T^2) p_i^2 x_j^2 + \frac{1}{2} T p_i p_j x_j^2 \right] \right]$

In[*]:= **Simplify** [$\frac{1}{4} (1 - T) p_i^2 x_i x_j - \frac{1}{2} p_i p_j x_i x_j + \frac{1}{4} (-T + T^2) p_i^2 x_j^2 + \frac{1}{2} T p_i p_j x_j^2$]

Out[*]:= $\frac{1}{4} p_i ((-1 + T) p_i + 2 p_j) x_j (-x_i + T x_j)$

In[*]:= **Simplify** [

$$\left(\mathbb{E} @ \Gamma \left[\omega, \{x_1, x_2, x_3\} \cdot \begin{pmatrix} \alpha & \beta & \gamma \\ \delta & \epsilon & \eta \\ \lambda & \mu & \nu \end{pmatrix} \cdot \{p_1, p_2, p_3\} \right] \mathbb{E}_{\{\} \rightarrow \{i,j\}} \left[1, \theta, \in \text{Series} \left[\theta, p_i^2 x_i x_j \right] \right] // m_{1,i \rightarrow 1} // m_{1,2 \rightarrow 1} // m_{1,j \rightarrow 1} // m_{1,3 \rightarrow 1} \right] \llbracket 3, 2 \rrbracket /. (x | p)_ \rightarrow \theta]$$

Out[*]:= $\frac{2 \gamma (\beta + \gamma \epsilon - \beta \eta)}{(-1 + \beta + \gamma \epsilon + \eta - \beta \eta)^2}$

In[*]:= **Simplify** [

$$\left(\mathbb{E} @ \Gamma \left[\omega, \{x_1, x_2, x_3\} \cdot \begin{pmatrix} \alpha & \beta & \gamma \\ \delta & \epsilon & \eta \\ \lambda & \mu & \nu \end{pmatrix} \cdot \{p_1, p_2, p_3\} \right] \mathbb{E}_{\{\} \rightarrow \{i,j\}} \left[1, \theta, \in \text{Series} \left[\theta, p_j x_j \right] \right] // m_{1,i \rightarrow 1} // m_{1,2 \rightarrow 1} // m_{1,j \rightarrow 1} // m_{1,3 \rightarrow 1} \right] \llbracket 3, 2 \rrbracket /. (x | p)_ \rightarrow \theta]$$

In[*]:= $\mathbb{E} @ \Gamma \left[\omega, \{x_1, x_2, x_3\} \cdot \begin{pmatrix} \alpha & \beta & \gamma \\ \delta & \epsilon & \eta \\ \lambda & \mu & \nu \end{pmatrix} \cdot \{p_1, p_2, p_3\} \right] \text{RP}_{i,j} // m_{1,i \rightarrow 1} // m_{1,2 \rightarrow 1} // m_{1,j \rightarrow 1} // m_{1,3 \rightarrow 1}$

Out[*]:= $\mathbb{E}_{\{\} \rightarrow \{1\}} \left[\frac{1}{\omega - \beta \omega - \gamma \epsilon \omega - \eta \omega + \beta \eta \omega}, \frac{(1 - \beta - \gamma \epsilon - \eta + \beta \eta - \lambda + \beta \lambda + \gamma \epsilon \lambda + \eta \lambda - \beta \eta \lambda - \alpha \mu - \gamma \delta \mu + \alpha \eta \mu - \delta \nu + \beta \delta \nu - \alpha \epsilon \nu) p_1 x_1}{1 - \beta - \gamma \epsilon - \eta + \beta \eta} \right]$
 $\in \text{Series} \left[\theta, (\beta \gamma - T \beta \gamma - T \gamma^2 + T^2 \gamma^2 - \gamma \epsilon - \beta \gamma \epsilon + \gamma^2 \epsilon + T \gamma^2 \epsilon - \gamma^2 \epsilon^2 - \beta \eta + \beta^2 \eta + 2 T \gamma \eta - \beta \gamma \eta - T \beta \gamma \eta - \gamma \epsilon \eta + 2 \beta \gamma \epsilon \eta + \beta \eta^2 - \beta^2 \eta^2) / (2 - 4 \beta + 2 \beta^2 - 4 \gamma \epsilon + 4 \beta \gamma \epsilon + 2 \gamma^2 \epsilon^2 - 4 \eta + 8 \beta \eta - 4 \beta^2 \eta + 4 \gamma \epsilon \eta - 4 \beta \gamma \epsilon \eta + 2 \eta^2 - 4 \beta \eta^2 + 2 \beta^2 \eta^2) + ((-\alpha \gamma \mu + T \alpha \gamma \mu - \alpha \beta \gamma \mu + T \alpha \beta \gamma \mu + 2 T \alpha \gamma^2 \mu - 2 T^2 \alpha \gamma^2 \mu + \gamma \delta \mu - \beta^2 \gamma \delta \mu - \gamma^2 \delta \mu - T \gamma^2 \delta \mu - \beta \gamma^2 \delta \mu + 3 T \beta \gamma^2 \delta \mu + 2 T \gamma^3 \delta \mu - 2 T^2 \gamma^3 \delta \mu + 3 \alpha \gamma \epsilon \mu + \alpha \beta \gamma \epsilon \mu - \alpha \gamma^2 \epsilon \mu - 3 T \alpha \gamma^2 \epsilon \mu + 3 \gamma^2 \delta \epsilon \mu - \beta \gamma^2 \delta \epsilon \mu - \gamma^3 \delta \epsilon \mu - T \gamma^3 \delta \epsilon \mu + \alpha \gamma^2 \epsilon^2 \mu + \alpha \eta \mu - \alpha \beta \eta \mu +$

$$\begin{aligned}
 & 2\alpha\gamma\eta\mu - 4T\alpha\gamma\eta\mu + 2\alpha\beta\gamma\eta\mu - 2T\alpha\gamma^2\eta\mu + 2T^2\alpha\gamma^2\eta\mu - \beta\gamma\delta\eta\mu + \beta^2\gamma\delta\eta\mu + \gamma^2\delta\eta\mu - \\
 & 3T\gamma^2\delta\eta\mu + \beta\gamma^2\delta\eta\mu + T\beta\gamma^2\delta\eta\mu - 4\alpha\gamma\epsilon\eta\mu - \alpha\beta\gamma\epsilon\eta\mu + \alpha\gamma^2\epsilon\eta\mu + T\alpha\gamma^2\epsilon\eta\mu - \\
 & \gamma^2\delta\epsilon\eta\mu - 2\alpha\eta^2\mu + 2\alpha\beta\eta^2\mu - \alpha\gamma\eta^2\mu + 3T\alpha\gamma\eta^2\mu - \alpha\beta\gamma\eta^2\mu - T\alpha\beta\gamma\eta^2\mu - \gamma\delta\eta^2\mu + \\
 & \beta\gamma\delta\eta^2\mu + \alpha\gamma\epsilon\eta^2\mu + \alpha\eta^3\mu - \alpha\beta\eta^3\mu - \alpha\beta\nu + T\alpha\beta\nu + \alpha\beta^2\nu - T\alpha\beta^2\nu + 2T\alpha\gamma\nu - \\
 & 2T^2\alpha\gamma\nu - 2T\alpha\beta\gamma\nu + 2T^2\alpha\beta\gamma\nu + \beta\delta\nu - 2\beta^2\delta\nu + \beta^3\delta\nu - 2T\gamma\delta\nu - \beta\gamma\delta\nu + 5T\beta\gamma\delta\nu + \\
 & \beta^2\gamma\delta\nu - 3T\beta^2\gamma\delta\nu + 2T\gamma^2\delta\nu - 2T^2\gamma^2\delta\nu - 2T\beta\gamma^2\delta\nu + 2T^2\beta\gamma^2\delta\nu + \alpha\epsilon\nu - \alpha\beta^2\epsilon\nu - \\
 & 2\alpha\gamma\epsilon\nu - 2T\alpha\gamma\epsilon\nu + \alpha\beta\gamma\epsilon\nu + 3T\alpha\beta\gamma\epsilon\nu + 3\gamma\delta\epsilon\nu - 4\beta\gamma\delta\epsilon\nu + \beta^2\gamma\delta\epsilon\nu - \\
 & 2\gamma^2\delta\epsilon\nu + \beta\gamma^2\delta\epsilon\nu + T\beta\gamma^2\delta\epsilon\nu + 3\alpha\gamma\epsilon^2\nu - \alpha\beta\gamma\epsilon^2\nu + \gamma^2\delta\epsilon^2\nu - 2T\alpha\eta\nu + 2\alpha\beta\eta\nu + \\
 & 2T\alpha\beta\eta\nu - 2\alpha\beta^2\eta\nu - 2T\alpha\gamma\eta\nu + 2T^2\alpha\gamma\eta\nu + 2T\alpha\beta\gamma\eta\nu - 2T^2\alpha\beta\gamma\eta\nu - \beta\delta\eta\nu + \\
 & 2\beta^2\delta\eta\nu - \beta^3\delta\eta\nu - 2T\gamma\delta\eta\nu + \beta\gamma\delta\eta\nu + 3T\beta\gamma\delta\eta\nu - \beta^2\gamma\delta\eta\nu - T\beta^2\gamma\delta\eta\nu - \alpha\beta\epsilon\eta\nu + \\
 & \alpha\beta^2\epsilon\eta\nu + 2\alpha\gamma\epsilon\eta\nu - \alpha\beta\gamma\epsilon\eta\nu - T\alpha\beta\gamma\epsilon\eta\nu + \gamma\delta\epsilon\eta\nu - \beta\gamma\delta\epsilon\eta\nu - \alpha\gamma\epsilon^2\eta\nu + \\
 & 2T\alpha\eta^2\nu - \alpha\beta\eta^2\nu - 3T\alpha\beta\eta^2\nu + \alpha\beta^2\eta^2\nu + T\alpha\beta^2\eta^2\nu - \alpha\epsilon\eta^2\nu + \alpha\beta\epsilon\eta^2\nu) \mathbf{p_1 x_1} / \\
 & (2 - 6\beta + 6\beta^2 - 2\beta^3 - 6\gamma\epsilon + 12\beta\gamma\epsilon - 6\beta^2\gamma\epsilon + 6\gamma^2\epsilon^2 - 6\beta\gamma^2\epsilon^2 - 2\gamma^3\epsilon^3 - 6\eta + 18\beta\eta - \\
 & 18\beta^2\eta + 6\beta^3\eta + 12\gamma\epsilon\eta - 24\beta\gamma\epsilon\eta + 12\beta^2\gamma\epsilon\eta - 6\gamma^2\epsilon^2\eta + 6\beta\gamma^2\epsilon^2\eta + 6\eta^2 - 18\beta\eta^2 + \\
 & 18\beta^2\eta^2 - 6\beta^3\eta^2 - 6\gamma\epsilon\eta^2 + 12\beta\gamma\epsilon\eta^2 - 6\beta^2\gamma\epsilon\eta^2 - 2\eta^3 + 6\beta\eta^3 - 6\beta^2\eta^3 + 2\beta^3\eta^3) + \\
 & ((\alpha^2\gamma\mu^2 - T\alpha^2\gamma\mu^2 - T\alpha^2\gamma^2\mu^2 + T^2\alpha^2\gamma^2\mu^2 - 2\alpha\gamma\delta\mu^2 + 2\alpha\beta\gamma\delta\mu^2 + 2\alpha\gamma^2\delta\mu^2 - 2T\alpha\beta\gamma^2\delta\mu^2 - \\
 & 2T\alpha\gamma^3\delta\mu^2 + 2T^2\alpha\gamma^3\delta\mu^2 - 2\gamma^2\delta^2\mu^2 + 2\beta\gamma^2\delta^2\mu^2 + \gamma^3\delta^2\mu^2 + T\gamma^3\delta^2\mu^2 - 2T\beta\gamma^3\delta^2\mu^2 - \\
 & T\gamma^4\delta^2\mu^2 + T^2\gamma^4\delta^2\mu^2 - 2\alpha^2\gamma\epsilon\mu^2 + 2T\alpha^2\gamma^2\epsilon\mu^2 - 2\alpha\gamma^2\delta\epsilon\mu^2 + 2T\alpha\gamma^3\delta\epsilon\mu^2 - 3\alpha^2\gamma\eta\mu^2 + \\
 & 3T\alpha^2\gamma\eta\mu^2 + 2T\alpha^2\gamma^2\eta\mu^2 - 2T^2\alpha^2\gamma^2\eta\mu^2 + 4\alpha\gamma\delta\eta\mu^2 - 4\alpha\beta\gamma\delta\eta\mu^2 - 4\alpha\gamma^2\delta\eta\mu^2 + \\
 & 2T\alpha\gamma^2\delta\eta\mu^2 + 2T\alpha\beta\gamma^2\delta\eta\mu^2 + 2T\alpha\gamma^3\delta\eta\mu^2 - 2T^2\alpha\gamma^3\delta\eta\mu^2 + 2\gamma^2\delta^2\eta\mu^2 - 2\beta\gamma^2\delta^2\eta\mu^2 - \\
 & \gamma^3\delta^2\eta\mu^2 + T\gamma^3\delta^2\eta\mu^2 + 4\alpha^2\gamma\epsilon\eta\mu^2 - 2T\alpha^2\gamma^2\epsilon\eta\mu^2 + 2\alpha\gamma^2\delta\epsilon\eta\mu^2 + 3\alpha^2\gamma\eta^2\mu^2 - \\
 & 3T\alpha^2\gamma\eta^2\mu^2 - T\alpha^2\gamma^2\eta^2\mu^2 + T^2\alpha^2\gamma^2\eta^2\mu^2 - 2\alpha\gamma\delta\eta^2\mu^2 + 2\alpha\beta\gamma\delta\eta^2\mu^2 + 2\alpha\gamma^2\delta\eta^2\mu^2 - \\
 & 2T\alpha\gamma^2\delta\eta^2\mu^2 - 2\alpha^2\gamma\epsilon\eta^2\mu^2 - \alpha^2\gamma\eta^3\mu^2 + T\alpha^2\gamma\eta^3\mu^2 + \alpha^2\mu\nu - T\alpha^2\mu\nu - \alpha^2\beta\mu\nu + \\
 & T\alpha^2\beta\mu\nu - 2T\alpha^2\gamma\mu\nu + 2T^2\alpha^2\gamma\mu\nu + 2T\alpha^2\beta\gamma\mu\nu - 2T^2\alpha^2\beta\gamma\mu\nu - 2\alpha\delta\mu\nu + 4\alpha\beta\delta\mu\nu - \\
 & 2\alpha\beta^2\delta\mu\nu + 2\alpha\gamma\delta\mu\nu + 2T\alpha\gamma\delta\mu\nu - 2\alpha\beta\gamma\delta\mu\nu - 6T\alpha\beta\gamma\delta\mu\nu + 4T\alpha\beta^2\gamma\delta\mu\nu - \\
 & 4T\alpha\gamma^2\delta\mu\nu + 4T^2\alpha\gamma^2\delta\mu\nu + 4T\alpha\beta\gamma^2\delta\mu\nu - 4T^2\alpha\beta\gamma^2\delta\mu\nu - 2\gamma\delta^2\mu\nu + 4\beta\gamma\delta^2\mu\nu - \\
 & 2\beta^2\gamma\delta^2\mu\nu + \gamma^2\delta^2\mu\nu + 3T\gamma^2\delta^2\mu\nu - \beta\gamma^2\delta^2\mu\nu - 7T\beta\gamma^2\delta^2\mu\nu + 4T\beta^2\gamma^2\delta^2\mu\nu - \\
 & 2T\gamma^3\delta^2\mu\nu + 2T^2\gamma^3\delta^2\mu\nu + 2T\beta\gamma^3\delta^2\mu\nu - 2T^2\beta\gamma^3\delta^2\mu\nu - 2\alpha^2\epsilon\mu\nu + 2\alpha^2\beta\epsilon\mu\nu + \\
 & \alpha^2\gamma\epsilon\mu\nu + 3T\alpha^2\gamma\epsilon\mu\nu - 4T\alpha^2\beta\gamma\epsilon\mu\nu - 4\alpha\gamma\delta\epsilon\mu\nu + 4\alpha\beta\gamma\delta\epsilon\mu\nu + 2\alpha\gamma^2\delta\epsilon\mu\nu + \\
 & 2T\alpha\gamma^2\delta\epsilon\mu\nu - 4T\alpha\beta\gamma^2\delta\epsilon\mu\nu - 2\gamma^2\delta^2\epsilon\mu\nu + 2\beta\gamma^2\delta^2\epsilon\mu\nu + \gamma^3\delta^2\epsilon\mu\nu - T\gamma^3\delta^2\epsilon\mu\nu - \\
 & 2\alpha^2\gamma\epsilon^2\mu\nu - 2\alpha\gamma^2\delta\epsilon^2\mu\nu - 3\alpha^2\eta\mu\nu + 3T\alpha^2\eta\mu\nu + 3\alpha^2\beta\eta\mu\nu - 3T\alpha^2\beta\eta\mu\nu + \\
 & 4T\alpha^2\gamma\eta\mu\nu - 4T^2\alpha^2\gamma\eta\mu\nu - 4T\alpha^2\beta\gamma\eta\mu\nu + 4T^2\alpha^2\beta\gamma\eta\mu\nu + 4\alpha\delta\eta\mu\nu - 8\alpha\beta\delta\eta\mu\nu + \\
 & 4\alpha\beta^2\delta\eta\mu\nu - 4\alpha\gamma\delta\eta\mu\nu + 4\alpha\beta\gamma\delta\eta\mu\nu + 4T\alpha\beta\gamma\delta\eta\mu\nu - 4T\alpha\beta^2\gamma\delta\eta\mu\nu + \\
 & 4T\alpha\gamma^2\delta\eta\mu\nu - 4T^2\alpha\gamma^2\delta\eta\mu\nu - 4T\alpha\beta\gamma^2\delta\eta\mu\nu + 4T^2\alpha\beta\gamma^2\delta\eta\mu\nu + 2\gamma\delta^2\eta\mu\nu - \\
 & 4\beta\gamma\delta^2\eta\mu\nu + 2\beta^2\gamma\delta^2\eta\mu\nu - \gamma^2\delta^2\eta\mu\nu + T\gamma^2\delta^2\eta\mu\nu + \beta\gamma^2\delta^2\eta\mu\nu - T\beta\gamma^2\delta^2\eta\mu\nu + \\
 & 4\alpha^2\epsilon\eta\mu\nu - 4\alpha^2\beta\epsilon\eta\mu\nu - 2\alpha^2\gamma\epsilon\eta\mu\nu - 2T\alpha^2\gamma\epsilon\eta\mu\nu + 4T\alpha^2\beta\gamma\epsilon\eta\mu\nu + 4\alpha\gamma\delta\epsilon\eta\mu\nu - \\
 & 4\alpha\beta\gamma\delta\epsilon\eta\mu\nu - 2\alpha\gamma^2\delta\epsilon\eta\mu\nu + 2T\alpha\gamma^2\delta\epsilon\eta\mu\nu + 2\alpha^2\gamma\epsilon^2\eta\mu\nu + 3\alpha^2\eta^2\mu\nu - \\
 & 3T\alpha^2\eta^2\mu\nu - 3\alpha^2\beta\eta^2\mu\nu + 3T\alpha^2\beta\eta^2\mu\nu - 2T\alpha^2\gamma\eta^2\mu\nu + 2T^2\alpha^2\gamma\eta^2\mu\nu + 2T\alpha^2\beta\gamma\eta^2\mu\nu - \\
 & 2T^2\alpha^2\beta\gamma\eta^2\mu\nu - 2\alpha\delta\eta^2\mu\nu + 4\alpha\beta\delta\eta^2\mu\nu - 2\alpha\beta^2\delta\eta^2\mu\nu + 2\alpha\gamma\delta\eta^2\mu\nu - 2T\alpha\gamma\delta\eta^2\mu\nu - \\
 & 2\alpha\beta\gamma\delta\eta^2\mu\nu + 2T\alpha\beta\gamma\delta\eta^2\mu\nu - 2\alpha^2\epsilon\eta^2\mu\nu + 2\alpha^2\beta\epsilon\eta^2\mu\nu + \alpha^2\gamma\epsilon\eta^2\mu\nu - \\
 & T\alpha^2\gamma\epsilon\eta^2\mu\nu - \alpha^2\eta^3\mu\nu + T\alpha^2\eta^3\mu\nu + \alpha^2\beta\eta^3\mu\nu - T\alpha^2\beta\eta^3\mu\nu - T\alpha^2\nu^2 + T^2\alpha^2\nu^2 + \\
 & 2T\alpha^2\beta\nu^2 - 2T^2\alpha^2\beta\nu^2 - T\alpha^2\beta^2\nu^2 + T^2\alpha^2\beta^2\nu^2 + 2T\alpha\delta\nu^2 - 6T\alpha\beta\delta\nu^2 + 6T\alpha\beta^2\delta\nu^2 - \\
 & 2T\alpha\beta^3\delta\nu^2 - 2T\alpha\gamma\delta\nu^2 + 2T^2\alpha\gamma\delta\nu^2 + 4T\alpha\beta\gamma\delta\nu^2 - 4T^2\alpha\beta\gamma\delta\nu^2 - 2T\alpha\beta^2\gamma\delta\nu^2 + \\
 & 2T^2\alpha\beta^2\gamma\delta\nu^2 + 2T\gamma\delta^2\nu^2 - 6T\beta\gamma\delta^2\nu^2 + 6T\beta^2\gamma\delta^2\nu^2 - 2T\beta^3\gamma\delta^2\nu^2 - T\gamma^2\delta^2\nu^2 +
 \end{aligned}$$

$$\begin{aligned} & T^2 \gamma^2 \delta^2 v^2 + 2 T \beta \gamma^2 \delta^2 v^2 - 2 T^2 \beta \gamma^2 \delta^2 v^2 - T \beta^2 \gamma^2 \delta^2 v^2 + T^2 \beta^2 \gamma^2 \delta^2 v^2 + \alpha^2 \epsilon v^2 + T \alpha^2 \epsilon v^2 - \\ & \alpha^2 \beta \epsilon v^2 - 3 T \alpha^2 \beta \epsilon v^2 + 2 T \alpha^2 \beta^2 \epsilon v^2 - 2 \alpha \delta \epsilon v^2 + 4 \alpha \beta \delta \epsilon v^2 - 2 \alpha \beta^2 \delta \epsilon v^2 + 2 \alpha \gamma \delta \epsilon v^2 - \\ & 2 \alpha \beta \gamma \delta \epsilon v^2 - 2 T \alpha \beta \gamma \delta \epsilon v^2 + 2 T \alpha \beta^2 \gamma \delta \epsilon v^2 - 2 \gamma \delta^2 \epsilon v^2 + 4 \beta \gamma \delta^2 \epsilon v^2 - 2 \beta^2 \gamma \delta^2 \epsilon v^2 + \\ & \gamma^2 \delta^2 \epsilon v^2 - T \gamma^2 \delta^2 \epsilon v^2 - \beta \gamma^2 \delta^2 \epsilon v^2 + T \beta \gamma^2 \delta^2 \epsilon v^2 - 2 \alpha^2 \epsilon^2 v^2 + 2 \alpha^2 \beta \epsilon^2 v^2 - 2 \alpha \gamma \delta \epsilon^2 v^2 + \\ & 2 \alpha \beta \gamma \delta \epsilon^2 v^2 + 2 T \alpha^2 \eta v^2 - 2 T^2 \alpha^2 \eta v^2 - 4 T \alpha^2 \beta \eta v^2 + 4 T^2 \alpha^2 \beta \eta v^2 + 2 T \alpha^2 \beta^2 \eta v^2 - \\ & 2 T^2 \alpha^2 \beta^2 \eta v^2 - 2 T \alpha \delta \eta v^2 + 6 T \alpha \beta \delta \eta v^2 - 6 T \alpha \beta^2 \delta \eta v^2 + 2 T \alpha \beta^3 \delta \eta v^2 + 2 T \alpha \gamma \delta \eta v^2 - \\ & 2 T^2 \alpha \gamma \delta \eta v^2 - 4 T \alpha \beta \gamma \delta \eta v^2 + 4 T^2 \alpha \beta \gamma \delta \eta v^2 + 2 T \alpha \beta^2 \gamma \delta \eta v^2 - 2 T^2 \alpha \beta^2 \gamma \delta \eta v^2 - \\ & 2 \alpha^2 \epsilon \eta v^2 + 2 \alpha^2 \beta \epsilon \eta v^2 + 2 T \alpha^2 \beta \epsilon \eta v^2 - 2 T \alpha^2 \beta^2 \epsilon \eta v^2 + 2 \alpha \delta \epsilon \eta v^2 - 4 \alpha \beta \delta \epsilon \eta v^2 + \\ & 2 \alpha \beta^2 \delta \epsilon \eta v^2 - 2 \alpha \gamma \delta \epsilon \eta v^2 + 2 T \alpha \gamma \delta \epsilon \eta v^2 + 2 \alpha \beta \gamma \delta \epsilon \eta v^2 - 2 T \alpha \beta \gamma \delta \epsilon \eta v^2 + \\ & 2 \alpha^2 \epsilon^2 \eta v^2 - 2 \alpha^2 \beta \epsilon^2 \eta v^2 - T \alpha^2 \eta^2 v^2 + T^2 \alpha^2 \eta^2 v^2 + 2 T \alpha^2 \beta \eta^2 v^2 - 2 T^2 \alpha^2 \beta \eta^2 v^2 - \\ & T \alpha^2 \beta^2 \eta^2 v^2 + T^2 \alpha^2 \beta^2 \eta^2 v^2 + \alpha^2 \epsilon \eta^2 v^2 - T \alpha^2 \epsilon \eta^2 v^2 - \alpha^2 \beta \epsilon \eta^2 v^2 + T \alpha^2 \beta \epsilon \eta^2 v^2) \mathbf{p}_1^2 \mathbf{x}_1^2) / \\ & (4 - 16 \beta + 24 \beta^2 - 16 \beta^3 + 4 \beta^4 - 16 \gamma \epsilon + 48 \beta \gamma \epsilon - 48 \beta^2 \gamma \epsilon + 16 \beta^3 \gamma \epsilon + 24 \gamma^2 \epsilon^2 - \\ & 48 \beta \gamma^2 \epsilon^2 + 24 \beta^2 \gamma^2 \epsilon^2 - 16 \gamma^3 \epsilon^3 + 16 \beta \gamma^3 \epsilon^3 + 4 \gamma^4 \epsilon^4 - 16 \eta + 64 \beta \eta - \\ & 96 \beta^2 \eta + 64 \beta^3 \eta - 16 \beta^4 \eta + 48 \gamma \epsilon \eta - 144 \beta \gamma \epsilon \eta + 144 \beta^2 \gamma \epsilon \eta - 48 \beta^3 \gamma \epsilon \eta - \\ & 48 \gamma^2 \epsilon^2 \eta + 96 \beta \gamma^2 \epsilon^2 \eta - 48 \beta^2 \gamma^2 \epsilon^2 \eta + 16 \gamma^3 \epsilon^3 \eta - 16 \beta \gamma^3 \epsilon^3 \eta + \\ & 24 \eta^2 - 96 \beta \eta^2 + 144 \beta^2 \eta^2 - 96 \beta^3 \eta^2 + 24 \beta^4 \eta^2 - 48 \gamma \epsilon \eta^2 + 144 \beta \gamma \epsilon \eta^2 - \\ & 144 \beta^2 \gamma \epsilon \eta^2 + 48 \beta^3 \gamma \epsilon \eta^2 + 24 \gamma^2 \epsilon^2 \eta^2 - 48 \beta \gamma^2 \epsilon^2 \eta^2 + 24 \beta^2 \gamma^2 \epsilon^2 \eta^2 - \\ & 16 \eta^3 + 64 \beta \eta^3 - 96 \beta^2 \eta^3 + 64 \beta^3 \eta^3 - 16 \beta^4 \eta^3 + 16 \gamma \epsilon \eta^3 - 48 \beta \gamma \epsilon \eta^3 + \\ & 48 \beta^2 \gamma \epsilon \eta^3 - 16 \beta^3 \gamma \epsilon \eta^3 + 4 \eta^4 - 16 \beta \eta^4 + 24 \beta^2 \eta^4 - 16 \beta^3 \eta^4 + 4 \beta^4 \eta^4) \end{aligned}$$

$$\begin{aligned} \text{In}[*]:= & \text{Simplify} \left[(\beta \gamma - T \beta \gamma - T \gamma^2 + T^2 \gamma^2 - \gamma \epsilon - \beta \gamma \epsilon + \gamma^2 \epsilon + T \gamma^2 \epsilon - \right. \\ & \left. \gamma^2 \epsilon^2 - \beta \eta + \beta^2 \eta + 2 T \gamma \eta - \beta \gamma \eta - T \beta \gamma \eta - \gamma \epsilon \eta + 2 \beta \gamma \epsilon \eta + \beta \eta^2 - \beta^2 \eta^2) / \right. \\ & \left. (2 - 4 \beta + 2 \beta^2 - 4 \gamma \epsilon + 4 \beta \gamma \epsilon + 2 \gamma^2 \epsilon^2 - 4 \eta + 8 \beta \eta - 4 \beta^2 \eta + 4 \gamma \epsilon \eta - 4 \beta \gamma \epsilon \eta + 2 \eta^2 - 4 \beta \eta^2 + 2 \beta^2 \eta^2) \right] \end{aligned}$$

$$\begin{aligned} \text{Out}[*]:= & (\beta (-1 + \eta) \eta - \beta^2 (-1 + \eta) \eta - \beta \gamma (-1 + T + \epsilon + \eta + T \eta - 2 \epsilon \eta) + \\ & \gamma (T^2 \gamma - \epsilon (1 + \gamma (-1 + \epsilon) + \eta) + T (\gamma (-1 + \epsilon) + 2 \eta))) / (2 (-1 + \beta + \gamma \epsilon + \eta - \beta \eta)^2) \end{aligned}$$

$$\text{In}[*]:= \left(\text{IdentityMatrix}[3] - \begin{pmatrix} \alpha & \beta & \gamma \\ \delta & \epsilon & \eta \\ \lambda & \mu & \nu \end{pmatrix} \right) // \text{Det} // \text{Simplify}$$

$$\text{Out}[*]:= 1 - \epsilon - \gamma \lambda + \gamma \epsilon \lambda - \gamma \delta \mu - \eta \mu - \nu + \epsilon \nu - \beta (\delta + \eta \lambda - \delta \nu) + \alpha (-1 + \epsilon + \eta \mu + \nu - \epsilon \nu)$$

Solving for R, C, \$k = 2

$$\begin{aligned} \text{In}[*]:= & \mathbf{\$k = 2;} \\ & \{\mathbf{R_{1,2}}, \mathbf{C_1}\} \\ & \text{Short}[\mathbf{RMoves}, \mathbf{20}] \end{aligned}$$

$$\begin{aligned}
 \text{Out}[*]= & \left\{ \mathbb{E}_{\{\} \rightarrow \{1,2\}} \left[\sqrt{T}, (-1+T) p_1 x_2 + (1-T) p_2 x_2, \right. \right. \\
 & \in \text{Series} \left[\theta, \frac{1}{4} (-1+T) p_1^2 x_1 x_2 - \frac{1}{2} T p_1 p_2 x_1 x_2 + \frac{1}{4} (T-T^2) p_1^2 x_2^2 + \frac{1}{2} T^2 p_1 p_2 x_2^2, \right. \\
 & c_{2,1} + T p_2 x_1 c_{2,4} + p_1 x_1 (c_{2,2} + c_{2,4} - T c_{2,4}) + T p_2 x_2 c_{2,5} + p_1 x_2 (c_{2,3} + c_{2,5} - T c_{2,5}) + T^2 p_2^2 x_1^2 c_{2,12} + \\
 & p_1 p_2 x_1^2 (T c_{2,9} + 2 T c_{2,12} - 2 T^2 c_{2,12}) + p_1^2 x_1^2 (c_{2,6} + c_{2,9} - T c_{2,9} + c_{2,12} - 2 T c_{2,12} + T^2 c_{2,12}) + \\
 & T^2 p_2^2 x_1 x_2 c_{2,13} + p_1 p_2 x_1 x_2 (T c_{2,10} + 2 T c_{2,13} - 2 T^2 c_{2,13}) + \\
 & p_1^2 x_1 x_2 (c_{2,7} + c_{2,10} - T c_{2,10} + c_{2,13} - 2 T c_{2,13} + T^2 c_{2,13}) + T^2 p_2^2 x_2^2 c_{2,14} + \\
 & p_1 p_2 x_2^2 (T c_{2,11} + 2 T c_{2,14} - 2 T^2 c_{2,14}) + p_1^2 x_2^2 (c_{2,8} + c_{2,11} - T c_{2,11} + c_{2,14} - 2 T c_{2,14} + T^2 c_{2,14}) + \\
 & T^3 p_2^3 x_1^3 c_{2,27} + p_1 p_2^2 x_1^3 (T^2 c_{2,23} + 3 T^2 c_{2,27} - 3 T^3 c_{2,27}) + \\
 & p_1^3 x_1^3 (c_{2,15} + c_{2,19} - T c_{2,19} + c_{2,23} - 2 T c_{2,23} + T^2 c_{2,23} + c_{2,27} - 3 T c_{2,27} + 3 T^2 c_{2,27} - T^3 c_{2,27}) + \\
 & p_1^2 p_2 x_1^3 (T c_{2,19} + 2 T c_{2,23} - 2 T^2 c_{2,23} + 3 T c_{2,27} - 6 T^2 c_{2,27} + 3 T^3 c_{2,27}) + \\
 & T^3 p_2^3 x_1^2 x_2 c_{2,28} + p_1 p_2^2 x_1^2 x_2 (T^2 c_{2,24} + 3 T^2 c_{2,28} - 3 T^3 c_{2,28}) + \\
 & p_1^3 x_1^2 x_2 (c_{2,16} + c_{2,20} - T c_{2,20} + c_{2,24} - 2 T c_{2,24} + T^2 c_{2,24} + c_{2,28} - 3 T c_{2,28} + 3 T^2 c_{2,28} - T^3 c_{2,28}) + \\
 & p_1^2 p_2 x_1^2 x_2 (T c_{2,20} + 2 T c_{2,24} - 2 T^2 c_{2,24} + 3 T c_{2,28} - 6 T^2 c_{2,28} + 3 T^3 c_{2,28}) + \\
 & T^3 p_2^3 x_1 x_2^2 c_{2,29} + p_1 p_2^2 x_1 x_2^2 (T^2 c_{2,25} + 3 T^2 c_{2,29} - 3 T^3 c_{2,29}) + \\
 & p_1^3 x_1 x_2^2 (c_{2,17} + c_{2,21} - T c_{2,21} + c_{2,25} - 2 T c_{2,25} + T^2 c_{2,25} + c_{2,29} - 3 T c_{2,29} + 3 T^2 c_{2,29} - T^3 c_{2,29}) + \\
 & p_1^2 p_2 x_1 x_2^2 (T c_{2,21} + 2 T c_{2,25} - 2 T^2 c_{2,25} + 3 T c_{2,29} - 6 T^2 c_{2,29} + 3 T^3 c_{2,29}) + \\
 & T^3 p_2^3 x_2^3 c_{2,30} + p_1 p_2^2 x_2^3 (T^2 c_{2,26} + 3 T^2 c_{2,30} - 3 T^3 c_{2,30}) + \\
 & p_1^3 x_2^3 (c_{2,18} + c_{2,22} - T c_{2,22} + c_{2,26} - 2 T c_{2,26} + T^2 c_{2,26} + c_{2,30} - 3 T c_{2,30} + 3 T^2 c_{2,30} - T^3 c_{2,30}) + \\
 & p_1^2 p_2 x_2^3 (T c_{2,22} + 2 T c_{2,26} - 2 T^2 c_{2,26} + 3 T c_{2,30} - 6 T^2 c_{2,30} + 3 T^3 c_{2,30}) \left. \right], \\
 & \mathbb{E}_{\{\} \rightarrow \{1\}} \left[\sqrt{T}, \theta, \in \text{Series} \left[\theta, \frac{p_1 x_1}{2}, e_{2,1} + p_1 x_1 e_{2,2} + p_1^2 x_1^2 e_{2,3} + p_1^3 x_1^3 e_{2,4} \right] \right] \left. \right\}
 \end{aligned}$$

Out[*]:=Short=

$$\left\{ \frac{1}{2} (-T^2 + T^3) p_1^2 p_3 x_1 x_2 x_3 - \frac{3}{4} T^3 p_1 p_2 p_3 x_1 x_2 x_3 + \right.$$

$$\frac{1}{4} (-T^3 + 3 T^4) p_1 p_2 p_3 x_1 x_3^2 + 3 C_{2,1} + T p_2 x_1 C_{2,4} + T p_3 x_1 C_{2,4} + T^2 p_3 x_2 C_{2,4} + \langle\langle 104 \rangle\rangle +$$

$$p_1 p_3^2 x_3^3 (T^4 C_{2,26} + 3 T^4 C_{2,30} - 3 T^6 C_{2,30}) + p_2 p_3^2 x_3^3 (T^5 C_{2,26} + 3 T^5 C_{2,30} - 3 T^6 C_{2,30}) +$$

$$p_2^3 x_3^3 (T^3 C_{2,18} + T^3 C_{2,22} - T^4 C_{2,22} + T^3 C_{2,26} - 2 T^4 C_{2,26} + T^5 C_{2,26} + T^6 C_{2,27} -$$

$$3 T^7 C_{2,27} + 3 T^8 C_{2,27} - T^9 C_{2,27} + T^5 C_{2,28} - 3 T^6 C_{2,28} + 3 T^7 C_{2,28} - T^8 C_{2,28} + T^4 C_{2,29} -$$

$$3 T^5 C_{2,29} + 3 T^6 C_{2,29} - T^7 C_{2,29} + 2 T^3 C_{2,30} - 6 T^4 C_{2,30} + 6 T^5 C_{2,30} - 2 T^6 C_{2,30}) +$$

$$p_1 p_2^2 x_3^3 (3 T^2 C_{2,18} - 3 T^3 C_{2,18} + 3 T^2 C_{2,22} - 5 T^3 C_{2,22} + 2 T^4 C_{2,22} + T^5 C_{2,23} - 3 T^6 C_{2,23} + 3 T^7 C_{2,23} -$$

$$T^8 C_{2,23} + T^4 C_{2,24} - 3 T^5 C_{2,24} + 3 T^6 C_{2,24} - T^7 C_{2,24} + T^3 C_{2,25} - 3 T^4 C_{2,25} + 3 T^5 C_{2,25} - T^6 C_{2,25} +$$

$$4 T^2 C_{2,26} - 10 T^3 C_{2,26} + 8 T^4 C_{2,26} - 2 T^5 C_{2,26} + 3 T^5 C_{2,27} - 12 T^6 C_{2,27} + 18 T^7 C_{2,27} - 12 T^8 C_{2,27} +$$

$$3 T^9 C_{2,27} + 3 T^4 C_{2,28} - 12 T^5 C_{2,28} + 18 T^6 C_{2,28} - 12 T^7 C_{2,28} + 3 T^8 C_{2,28} + 3 T^3 C_{2,29} - 12 T^4 C_{2,29} +$$

$$18 T^5 C_{2,29} - 12 T^6 C_{2,29} + 3 T^7 C_{2,29} + 6 T^2 C_{2,30} - 21 T^3 C_{2,30} + 27 T^4 C_{2,30} - 15 T^5 C_{2,30} + 3 T^6 C_{2,30}) +$$

$$p_2^2 p_3 x_3^3 (T^4 C_{2,22} + 2 T^4 C_{2,26} - 2 T^5 C_{2,26} + 3 T^4 C_{2,30} - 6 T^5 C_{2,30} + 3 T^6 C_{2,30}) +$$

$$\frac{1}{2} p_1^2 p_3 x_3^3 (-T^5 + T^6 + 2 T^2 C_{2,22} - 4 T^3 C_{2,22} + 4 T^4 C_{2,22} + 4 T^2 C_{2,26} - 8 T^3 C_{2,26} + 8 T^4 C_{2,26} - 4 T^5 C_{2,26} +$$

$$6 T^2 C_{2,30} - 12 T^3 C_{2,30} + 12 T^4 C_{2,30} - 12 T^5 C_{2,30} + 6 T^6 C_{2,30}) = 3 C_{2,1} + T^2 p_3 x_1 C_{2,4} +$$

$$\langle\langle 143 \rangle\rangle + p_2^2 p_3 x_3^3 (T^4 C_{2,22} + 2 T^4 C_{2,26} - 2 T^5 C_{2,26} + 6 T^4 C_{2,30} - 12 T^5 C_{2,30} + 6 T^6 C_{2,30}),$$

$$\langle\langle 7 \rangle\rangle, \frac{p_1^3 x_1^3 (T^3 C_{2,15} + T^2 C_{2,16} + \langle\langle 13 \rangle\rangle + C_{2,30} + e_{2,4})}{T^3} +$$

$$\left. \frac{p_1^2 \langle\langle 1 \rangle\rangle (\langle\langle 50 \rangle\rangle + \langle\langle 1 \rangle\rangle)}{8 T^3} + \frac{\langle\langle 1 \rangle\rangle}{4 \langle\langle 1 \rangle\rangle} + \frac{\langle\langle 47 \rangle\rangle + 24 T^3 e_{2,4}}{4 T^3} = \emptyset \right\}$$

In[*]:= unknowns = Cases [{R_{1,2}, R̄_{1,2}, C₁, C̄₁}, (c | d | e | f)_{\$k,_, ∞}] // Union

Out[*]:= {C_{2,1}, C_{2,2}, C_{2,3}, C_{2,4}, C_{2,5}, C_{2,6}, C_{2,7}, C_{2,8}, C_{2,9}, C_{2,10}, C_{2,11}, C_{2,12}, C_{2,13}, C_{2,14},
 C_{2,15}, C_{2,16}, C_{2,17}, C_{2,18}, C_{2,19}, C_{2,20}, C_{2,21}, C_{2,22}, C_{2,23}, C_{2,24}, C_{2,25}, C_{2,26}, C_{2,27},
 C_{2,28}, C_{2,29}, C_{2,30}, d_{2,1}, d_{2,2}, d_{2,3}, d_{2,4}, d_{2,5}, d_{2,6}, d_{2,7}, d_{2,8}, d_{2,9}, d_{2,10}, d_{2,11},
 d_{2,12}, d_{2,13}, d_{2,14}, d_{2,15}, d_{2,16}, d_{2,17}, d_{2,18}, d_{2,19}, d_{2,20}, d_{2,21}, d_{2,22}, d_{2,23}, d_{2,24},
 d_{2,25}, d_{2,26}, d_{2,27}, d_{2,28}, d_{2,29}, d_{2,30}, e_{2,1}, e_{2,2}, e_{2,3}, e_{2,4}, f_{2,1}, f_{2,2}, f_{2,3}, f_{2,4}}

In[]:= Short [errors = CCF /@ Cases [RMoves, a_ == b_ => a - b], 25]

Out[]//Short=

$$\left\{ \frac{1}{8} \left(-2 T p_1^3 x_1 x_2 x_3 + 4 T^2 p_1^3 x_1 x_2 x_3 - 2 T^3 p_1^3 x_1 x_2 x_3 - 4 T^2 p_1^2 p_2 x_1 x_2 x_3 + 4 T^3 p_1^2 p_2 x_1 x_2 x_3 + \right. \right.$$

$$2 T^2 p_1 p_2^2 x_1 x_2 x_3 - 2 T^3 p_1 p_2^2 x_1 x_2 x_3 - 2 T^2 p_1^2 p_3 x_1 x_2 x_3 + 2 T^3 p_1^2 p_3 x_1 x_2 x_3 + T^2 p_1^3 x_2^2 x_3 -$$

$$2 T^3 p_1^3 x_2^2 x_3 + T^4 p_1^3 x_2^2 x_3 + 3 T^3 p_1^2 p_2 x_2^2 x_3 - 3 T^4 p_1^2 p_2 x_2^2 x_3 + T^2 p_1 p_2^2 x_2^2 x_3 - 3 T^3 p_1 p_2^2 x_2^2 x_3 +$$

$$2 T^4 p_1 p_2^2 x_2^2 x_3 + 2 T^3 p_1 p_2 p_3 x_2^2 x_3 - 2 T^4 p_1 p_2 p_3 x_2^2 x_3 + 2 T^2 p_1^3 x_1 x_3^2 - 4 T^3 p_1^3 x_1 x_3^2 + 2 T^4 p_1^3 x_1 x_3^2 +$$

$$T^2 p_1^2 p_2 x_1 x_3^2 + T^3 p_1^2 p_2 x_1 x_3^2 - 2 T^4 p_1^2 p_2 x_1 x_3^2 - T^2 p_1^2 p_3 x_1 x_3^2 + 5 T^3 p_1^2 p_3 x_1 x_3^2 - 4 T^4 p_1^2 p_3 x_1 x_3^2 -$$

$$2 T^3 p_1 p_2 p_3 x_1 x_3^2 + 2 T^4 p_1 p_2 p_3 x_1 x_3^2 + T^3 p_1^3 x_2 x_3^2 - 2 T^4 p_1^3 x_2 x_3^2 + T^5 p_1^3 x_2 x_3^2 - T^3 p_1^2 p_2 x_2 x_3^2 +$$

$$2 T^4 p_1^2 p_2 x_2 x_3^2 - T^5 p_1^2 p_2 x_2 x_3^2 - 3 T^3 p_1 p_2^2 x_2 x_3^2 + 3 T^4 p_1 p_2^2 x_2 x_3^2 + T^3 p_1^2 p_3 x_2 x_3^2 + \ll 1963 \gg +$$

$$72 T p_1^2 p_2 x_2 x_3^2 c_{2,30} - 288 T^2 p_1^2 p_2 x_2 x_3^2 c_{2,30} + 432 T^3 p_1^2 p_2 x_2 x_3^2 c_{2,30} - 288 T^4 p_1^2 p_2 x_2 x_3^2 c_{2,30} +$$

$$72 T^5 p_1^2 p_2 x_2 x_3^2 c_{2,30} + 72 T^2 p_1 p_2^2 x_2 x_3^2 c_{2,30} - 216 T^3 p_1 p_2^2 x_2 x_3^2 c_{2,30} + 216 T^4 p_1 p_2^2 x_2 x_3^2 c_{2,30} -$$

$$72 T^5 p_1 p_2^2 x_2 x_3^2 c_{2,30} + 24 T^3 p_2^3 x_2 x_3^2 c_{2,30} - 48 T^4 p_2^3 x_2 x_3^2 c_{2,30} + 24 T^5 p_2^3 x_2 x_3^2 c_{2,30} + 8 p_1^3 x_3^3 c_{2,30} +$$

$$48 T p_1^3 x_3^3 c_{2,30} + 120 T^2 p_1^3 x_3^3 c_{2,30} - 152 T^3 p_1^3 x_3^3 c_{2,30} + 96 T^4 p_1^3 x_3^3 c_{2,30} - 24 T^5 p_1^3 x_3^3 c_{2,30} +$$

$$24 T p_1^2 p_2 x_3^3 c_{2,30} - 120 T^2 p_1^2 p_2 x_3^3 c_{2,30} + 240 T^3 p_1^2 p_2 x_3^3 c_{2,30} - 240 T^4 p_1^2 p_2 x_3^3 c_{2,30} +$$

$$120 T^5 p_1^2 p_2 x_3^3 c_{2,30} - 24 T^6 p_1 p_2^2 x_3^3 c_{2,30} + 24 T^2 p_1 p_2^2 x_3^3 c_{2,30} - 96 T^3 p_1 p_2^2 x_3^3 c_{2,30} +$$

$$144 T^4 p_1 p_2^2 x_3^3 c_{2,30} - 96 T^5 p_1 p_2^2 x_3^3 c_{2,30} + 24 T^6 p_1 p_2^2 x_3^3 c_{2,30} + 24 T^4 p_1^2 p_3 x_3^3 c_{2,30} -$$

$$48 T^5 p_1^2 p_3 x_3^3 c_{2,30} + 24 T^6 p_1^2 p_3 x_3^3 c_{2,30} - 24 T^4 p_2^2 p_3 x_3^3 c_{2,30} + 48 T^5 p_2^2 p_3 x_3^3 c_{2,30} - 24 T^6 p_2^2 p_3 x_3^3 c_{2,30} +$$

$$24 T^5 p_1 p_3^2 x_3^3 c_{2,30} - 24 T^6 p_1 p_3^2 x_3^3 c_{2,30} - 24 T^5 p_2 p_3^2 x_3^3 c_{2,30} + 24 T^6 p_2 p_3^2 x_3^3 c_{2,30} \left. \right\},$$

$$\frac{-T^2 p_1^2 x_1 x_2 + \ll 638 \gg + \ll 1 \gg}{8 T^3}, \ll 5 \gg,$$

$$\frac{\ll 1 \gg}{8 T^3},$$

$$\left. \frac{2 T - 8 T^2 + \ll 167 \gg + 8 p_1^3 x_1^3 e_{2,4}}{8 T^3} \right\}$$

In[]:= Short [# , 10] & [eqns =

Thread[0 == Union@@ (CoefficientRules[# , {x1, x2, x3, p1, p2, p3}][[; , 2] & /@ errors)]]]

Out[]//Short=

$$\left\{ \theta == T c_{2,4} - T^2 c_{2,4}, \theta == -T c_{2,4} + T^2 c_{2,4}, \theta == T^2 c_{2,4} - T^3 c_{2,4}, \right.$$

$$\ll 213 \gg, \theta == -\frac{1}{4} T + \frac{T}{4} - c_{2,15} + 3 T c_{2,15} - 3 T^2 c_{2,15} + T^3 c_{2,15} + c_{2,16} - 2 T c_{2,16} +$$

$$T^2 c_{2,16} - c_{2,17} + T c_{2,17} + c_{2,18} + T^3 d_{2,18} - T^2 d_{2,22} + T^3 d_{2,22} + T d_{2,26} - 2 T^2 d_{2,26} +$$

$$T^3 d_{2,26} - d_{2,30} + 3 T d_{2,30} - 3 T^2 d_{2,30} + T^3 d_{2,30} - f_{2,4} + 3 T f_{2,4} - 3 T^2 f_{2,4} + T^3 f_{2,4},$$

$$\theta == \frac{5 T}{4} - \frac{T^2}{2} + d_{2,2} + T d_{2,3} + d_{2,4} + T d_{2,5} - 2 T d_{2,7} - 4 T^2 d_{2,8} + d_{2,10} - 2 T d_{2,10} + 2 T d_{2,11} -$$

$$4 T^2 d_{2,11} + 2 d_{2,13} - 2 T d_{2,13} + 4 T d_{2,14} - 4 T^2 d_{2,14} + 6 T^2 d_{2,17} + 18 T^3 d_{2,18} -$$

$$4 T d_{2,21} + 6 T^2 d_{2,21} - 12 T^2 d_{2,22} + 18 T^3 d_{2,22} + 2 d_{2,25} - 8 T d_{2,25} + 6 T^2 d_{2,25} +$$

$$6 T d_{2,26} - 24 T^2 d_{2,26} + 18 T^3 d_{2,26} + 6 d_{2,29} - 12 T d_{2,29} + 6 T^2 d_{2,29} + 18 T d_{2,30} -$$

$$36 T^2 d_{2,30} + 18 T^3 d_{2,30} + T f_{2,2} + 4 T f_{2,3} - 4 T^2 f_{2,3} + 18 T f_{2,4} - 36 T^2 f_{2,4} + 18 T^3 f_{2,4} \left. \right\}$$

In[]:= {sol} = Solve[eqns, unknowns]

Solve: Equations may not give solutions for all "solve" variables.

$$\text{Out[]} = \left\{ \left\{ \begin{array}{l} c_{2,1} \rightarrow -\frac{c_{2,2}}{2} - \frac{c_{2,5}}{2}, c_{2,3} \rightarrow -T c_{2,2} - c_{2,5}, c_{2,4} \rightarrow 0, c_{2,6} \rightarrow 0, c_{2,7} \rightarrow -\frac{1}{2} (1+T) c_{2,10} - c_{2,11}, \\ c_{2,8} \rightarrow -\frac{1}{2} (T - T^2) c_{2,10} - (1-T) c_{2,11}, c_{2,9} \rightarrow 0, c_{2,12} \rightarrow 0, c_{2,13} \rightarrow 0, c_{2,14} \rightarrow 0, c_{2,15} \rightarrow 0, \\ c_{2,16} \rightarrow \frac{2+T}{24} - c_{2,26}, c_{2,17} \rightarrow \frac{1}{24} (-1-T+2T^2) - (1-T) c_{2,26}, c_{2,18} \rightarrow \frac{1}{24} (T+2T^2-3T^3), \\ c_{2,19} \rightarrow 0, c_{2,20} \rightarrow -\frac{1}{8}, c_{2,21} \rightarrow \frac{1}{8} (1+2T), c_{2,22} \rightarrow -\frac{T^2}{4} - (1-T) c_{2,26}, c_{2,23} \rightarrow 0, c_{2,24} \rightarrow 0, \\ c_{2,25} \rightarrow -\frac{1}{8}, c_{2,27} \rightarrow 0, c_{2,28} \rightarrow 0, c_{2,29} \rightarrow 0, c_{2,30} \rightarrow 0, d_{2,1} \rightarrow \frac{c_{2,2}}{2} + \frac{c_{2,5}}{2}, d_{2,2} \rightarrow -c_{2,2}, \\ d_{2,3} \rightarrow \frac{c_{2,2}}{T} + c_{2,5}, d_{2,4} \rightarrow 0, d_{2,5} \rightarrow -c_{2,5}, d_{2,6} \rightarrow 0, d_{2,7} \rightarrow -\frac{-1+T}{8T} - \frac{(1-3T) c_{2,10}}{2T} + \frac{c_{2,11}}{T}, \\ d_{2,8} \rightarrow -\frac{1-T}{8T^2} - \frac{(-1+3T-2T^2) c_{2,10}}{2T^2} - \frac{(1-T) c_{2,11}}{T^2}, d_{2,9} \rightarrow 0, d_{2,10} \rightarrow \frac{1}{4} - c_{2,10}, \\ d_{2,11} \rightarrow -\frac{1}{4T} - \frac{(-1+T) c_{2,10}}{T} - \frac{c_{2,11}}{T}, d_{2,12} \rightarrow 0, d_{2,13} \rightarrow 0, d_{2,14} \rightarrow 0, d_{2,15} \rightarrow 0, \\ d_{2,16} \rightarrow -\frac{2+T}{24T} + \frac{c_{2,26}}{T}, d_{2,17} \rightarrow -\frac{-5+T+4T^2}{24T^2} - \frac{(1-T) c_{2,26}}{T^2}, d_{2,18} \rightarrow -\frac{3-2T-T^2}{24T^3}, d_{2,19} \rightarrow 0, \\ d_{2,20} \rightarrow -\frac{1}{8}, d_{2,21} \rightarrow -\frac{-2-T}{8T}, d_{2,22} \rightarrow -\frac{1+T^2}{8T^2} - \frac{(1-T) c_{2,26}}{T^2}, d_{2,23} \rightarrow 0, d_{2,24} \rightarrow 0, d_{2,25} \rightarrow -\frac{1}{8}, \\ d_{2,26} \rightarrow -\frac{-1-T}{8T} - \frac{c_{2,26}}{T}, d_{2,27} \rightarrow 0, d_{2,28} \rightarrow 0, d_{2,29} \rightarrow 0, d_{2,30} \rightarrow 0, e_{2,1} \rightarrow -\frac{c_{2,2}}{2} - \frac{c_{2,5}}{2}, \\ e_{2,2} \rightarrow -c_{2,10}, e_{2,3} \rightarrow 0, e_{2,4} \rightarrow 0, f_{2,1} \rightarrow \frac{c_{2,2}}{2} + \frac{c_{2,5}}{2}, f_{2,2} \rightarrow -\frac{1}{4} + c_{2,10}, f_{2,3} \rightarrow 0, f_{2,4} \rightarrow 0 \end{array} \right\} \right\}$$

In[]:= sol /. (a_ -> b_) := (a = b)

$$\text{Out[]} = \left\{ -\frac{c_{2,2}}{2} - \frac{c_{2,5}}{2}, -T c_{2,2} - c_{2,5}, 0, 0, -\frac{1}{2} (1+T) c_{2,10} - c_{2,11}, -\frac{1}{2} (T-T^2) c_{2,10} - (1-T) c_{2,11}, \right. \\
 0, 0, 0, 0, 0, \frac{2+T}{24} - c_{2,26}, \frac{1}{24} (-1-T+2T^2) - (1-T) c_{2,26}, \frac{1}{24} (T+2T^2-3T^3), 0, -\frac{1}{8}, \\
 \frac{1}{8} (1+2T), -\frac{T^2}{4} - (1-T) c_{2,26}, 0, 0, -\frac{1}{8}, 0, 0, 0, 0, \frac{c_{2,2}}{2} + \frac{c_{2,5}}{2}, -c_{2,2}, \frac{c_{2,2}}{T} + c_{2,5}, 0, \\
 -c_{2,5}, 0, -\frac{-1+T}{8T} - \frac{(1-3T) c_{2,10}}{2T} + \frac{c_{2,11}}{T}, -\frac{1-T}{8T^2} - \frac{(-1+3T-2T^2) c_{2,10}}{2T^2} - \frac{(1-T) c_{2,11}}{T^2}, \\
 0, \frac{1}{4} - c_{2,10}, -\frac{1}{4T} - \frac{(-1+T) c_{2,10}}{T} - \frac{c_{2,11}}{T}, 0, 0, 0, 0, -\frac{2+T}{24T} + \frac{c_{2,26}}{T}, \\
 -\frac{-5+T+4T^2}{24T^2} - \frac{(1-T) c_{2,26}}{T^2}, -\frac{3-2T-T^2}{24T^3}, 0, -\frac{1}{8}, -\frac{-2-T}{8T}, -\frac{1+T^2}{8T^2} - \frac{(1-T) c_{2,26}}{T^2}, 0, 0, \\
 \left. -\frac{1}{8}, -\frac{-1-T}{8T} - \frac{c_{2,26}}{T}, 0, 0, 0, 0, -\frac{c_{2,2}}{2} - \frac{c_{2,5}}{2}, -c_{2,10}, 0, 0, \frac{c_{2,2}}{2} + \frac{c_{2,5}}{2}, -\frac{1}{4} + c_{2,10}, 0, 0 \right\}$$

In[]:= Cases [{R1,2, R1,2, C1, C1}, (c | d | e | f) \$k,_, inf] // Union

Out[]:= {C2,2, C2,5, C2,10, C2,11, C2,26}

In[]:= {Z1, Z2} = ZF /@ {Knot[10, 106], Knot[12, NonAlternating, 369]}

KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

$$\text{Out[]} = \left\{ \mathbb{E}_{\{ \} \rightarrow \{1\}} \left[\frac{T^4}{1-4T+9T^2-15T^3+17T^4-15T^5+9T^6-4T^7+T^8}, \right. \right. \\
 0, \text{Series} \left[0, (3-20T+69T^2-161T^3+272T^4-328T^5+225T^6+92T^7- \right. \\
 548T^8+952T^9-1113T^{10}+980T^{11}-668T^{12}+349T^{13}-135T^{14}+36T^{15}-5T^{16}) / \\
 (2-16T+68T^2-204T^3+470T^4-872T^5+1338T^6-1720T^7+1870T^8-1720T^9+ \\
 1338T^{10}-872T^{11}+470T^{12}-204T^{13}+68T^{14}-16T^{15}+2T^{16}), \\
 (3-40T+264T^2-1128T^3+3437T^4-7552T^5+10297T^6+2304T^7-67324T^8+259472T^9- \\
 699066T^{10}+1539252T^{11}-2919131T^{12}+4882760T^{13}-7290870T^{14}+9779044T^{15}- \\
 11816854T^{16}+12877354T^{17}-12651386T^{18}+11191592T^{19}-8896165T^{20}+6336738T^{21}- \\
 4030390T^{22}+2278962T^{23}-1139320T^{24}+500046T^{25}-190857T^{26}+62504T^{27}-17215T^{28}+ \\
 3862T^{29}-668T^{30}+80T^{31}-5T^{32}-32c_{2,2}+480Tc_{2,2}-3696T^2c_{2,2}+19448T^3c_{2,2}- \\
 78192T^4c_{2,2}+254496T^5c_{2,2}-693960T^6c_{2,2}+1620216T^7c_{2,2}-3284656T^8c_{2,2}+ \\
 5831784T^9c_{2,2}-9103728T^{10}c_{2,2}+12483640T^{11}c_{2,2}-14922472T^{12}c_{2,2}+15241728T^{13}c_{2,2}- \\
 12646416T^{14}c_{2,2}+7205208T^{15}c_{2,2}-7205208T^{17}c_{2,2}+12646416T^{18}c_{2,2}-15241728T^{19}c_{2,2}+ \\
 14922472T^{20}c_{2,2}-12483640T^{21}c_{2,2}+9103728T^{22}c_{2,2}-5831784T^{23}c_{2,2}+3284656T^{24}c_{2,2}- \\
 1620216T^{25}c_{2,2}+693960T^{26}c_{2,2}-254496T^{27}c_{2,2}+78192T^{28}c_{2,2}-19448T^{29}c_{2,2}+ \\
 3696T^{30}c_{2,2}-480T^{31}c_{2,2}+32T^{32}c_{2,2}-32c_{2,5}+480Tc_{2,5}-3696T^2c_{2,5}+19448T^3c_{2,5}- \\
 78192T^4c_{2,5}+254496T^5c_{2,5}-693960T^6c_{2,5}+1620216T^7c_{2,5}-3284656T^8c_{2,5}+ \\
 5831784T^9c_{2,5}-9103728T^{10}c_{2,5}+12483640T^{11}c_{2,5}-14922472T^{12}c_{2,5}+15241728T^{13}c_{2,5}-
 \left. \left. \right. \right\}$$

$$\begin{aligned}
 & 12\,646\,416\,T^{14}c_{2,5} + 7\,205\,208\,T^{15}c_{2,5} - 7\,205\,208\,T^{17}c_{2,5} + 12\,646\,416\,T^{18}c_{2,5} - 15\,241\,728\,T^{19}c_{2,5} + \\
 & 14\,922\,472\,T^{20}c_{2,5} - 12\,483\,640\,T^{21}c_{2,5} + 9\,103\,728\,T^{22}c_{2,5} - 5\,831\,784\,T^{23}c_{2,5} + 3\,284\,656\,T^{24}c_{2,5} - \\
 & 1\,620\,216\,T^{25}c_{2,5} + 693\,960\,T^{26}c_{2,5} - 254\,496\,T^{27}c_{2,5} + 78\,192\,T^{28}c_{2,5} - 19\,448\,T^{29}c_{2,5} + \\
 & 3\,696\,T^{30}c_{2,5} - 480\,T^{31}c_{2,5} + 32\,T^{32}c_{2,5} - 24\,c_{2,10} + 352\,T\,c_{2,10} - 2\,648\,T^2\,c_{2,10} + 13\,592\,T^3\,c_{2,10} - \\
 & 53\,208\,T^4\,c_{2,10} + 168\,192\,T^5\,c_{2,10} - 443\,688\,T^6\,c_{2,10} + 995\,864\,T^7\,c_{2,10} - 1\,920\,832\,T^8\,c_{2,10} + \\
 & 3\,187\,896\,T^9\,c_{2,10} - 4\,506\,304\,T^{10}\,c_{2,10} + 5\,249\,976\,T^{11}\,c_{2,10} - 4\,549\,000\,T^{12}\,c_{2,10} + 1\,601\,312\,T^{13}\,c_{2,10} + \\
 & 3\,879\,856\,T^{14}\,c_{2,10} - 11\,310\,184\,T^{15}\,c_{2,10} + 19\,225\,392\,T^{16}\,c_{2,10} - 25\,720\,600\,T^{17}\,c_{2,10} + \\
 & 29\,172\,688\,T^{18}\,c_{2,10} - 28\,882\,144\,T^{19}\,c_{2,10} + 25\,295\,944\,T^{20}\,c_{2,10} - 19\,717\,304\,T^{21}\,c_{2,10} + \\
 & 13\,701\,152\,T^{22}\,c_{2,10} - 8\,475\,672\,T^{23}\,c_{2,10} + 4\,648\,480\,T^{24}\,c_{2,10} - 2\,244\,568\,T^{25}\,c_{2,10} + 944\,232\,T^{26}\,c_{2,10} - \\
 & 340\,800\,T^{27}\,c_{2,10} + 103\,176\,T^{28}\,c_{2,10} - 25\,304\,T^{29}\,c_{2,10} + 4\,744\,T^{30}\,c_{2,10} - 608\,T^{31}\,c_{2,10} + 40\,T^{32}\,c_{2,10} \Big) / \\
 & \left(8 - 128\,T + 1056\,T^2 - 5984\,T^3 + 26\,064\,T^4 - 92\,544\,T^5 + 277\,584\,T^6 - 720\,096\,T^7 + 1\,642\,328\,T^8 - \right. \\
 & 3\,332\,448\,T^9 + 6\,069\,152\,T^{10} - 9\,986\,912\,T^{11} + 14\,922\,472\,T^{12} - 20\,322\,304\,T^{13} + 25\,292\,832\,T^{14} - \\
 & 28\,820\,832\,T^{15} + 30\,099\,512\,T^{16} - 28\,820\,832\,T^{17} + 25\,292\,832\,T^{18} - 20\,322\,304\,T^{19} + \\
 & 14\,922\,472\,T^{20} - 9\,986\,912\,T^{21} + 6\,069\,152\,T^{22} - 3\,332\,448\,T^{23} + 1\,642\,328\,T^{24} - 720\,096\,T^{25} + \\
 & \left. 277\,584\,T^{26} - 92\,544\,T^{27} + 26\,064\,T^{28} - 5984\,T^{29} + 1056\,T^{30} - 128\,T^{31} + 8\,T^{32} \right) \Big] ,
 \end{aligned}$$

$$E_{\{\} \rightarrow \{1\}} \left[\frac{T^4}{1 - 4\,T + 9\,T^2 - 15\,T^3 + 17\,T^4 - 15\,T^5 + 9\,T^6 - 4\,T^7 + T^8} \right] ,$$

0,

eSeries [0,

$$(3 - 20\,T + 69\,T^2 - 161\,T^3 + 272\,T^4 - 328\,T^5 + 225\,T^6 + 92\,T^7 - 548\,T^8 +$$

$$952\,T^9 - 1113\,T^{10} + 980\,T^{11} - 668\,T^{12} + 349\,T^{13} - 135\,T^{14} + 36\,T^{15} - 5\,T^{16}) /$$

$$(2 - 16\,T + 68\,T^2 - 204\,T^3 + 470\,T^4 - 872\,T^5 + 1338\,T^6 - 1720\,T^7 + 1870\,T^8 - 1720\,T^9 +$$

$$1338\,T^{10} - 872\,T^{11} + 470\,T^{12} - 204\,T^{13} + 68\,T^{14} - 16\,T^{15} + 2\,T^{16}) ,$$

$$(3 - 40\,T + 264\,T^2 - 1120\,T^3 + 3333\,T^4 - 6896\,T^5 + 7641\,T^6 + 9944\,T^7 - 83\,404\,T^8 + 283\,088\,T^9 -$$

$$716\,082\,T^{10} + 1\,514\,140\,T^{11} - 2\,796\,883\,T^{12} + 4\,607\,952\,T^{13} - 6\,839\,214\,T^{14} + 9\,183\,044\,T^{15} -$$

$$11\,164\,950\,T^{16} + 12\,281\,354\,T^{17} - 12\,199\,730\,T^{18} + 10\,916\,784\,T^{19} - 8\,773\,917\,T^{20} + 6\,311\,626\,T^{21} -$$

$$4\,047\,406\,T^{22} + 2\,302\,578\,T^{23} - 1\,155\,400\,T^{24} + 507\,686\,T^{25} - 193\,513\,T^{26} + 63\,160\,T^{27} - 17\,319\,T^{28} +$$

$$3870\,T^{29} - 668\,T^{30} + 80\,T^{31} - 5\,T^{32} - 32\,c_{2,2} + 480\,T\,c_{2,2} - 3\,696\,T^2\,c_{2,2} + 19\,448\,T^3\,c_{2,2} -$$

$$78\,192\,T^4\,c_{2,2} + 254\,496\,T^5\,c_{2,2} - 693\,960\,T^6\,c_{2,2} + 1\,620\,216\,T^7\,c_{2,2} - 3\,284\,656\,T^8\,c_{2,2} +$$

$$5\,831\,784\,T^9\,c_{2,2} - 9\,103\,728\,T^{10}\,c_{2,2} + 12\,483\,640\,T^{11}\,c_{2,2} - 14\,922\,472\,T^{12}\,c_{2,2} + 15\,241\,728\,T^{13}\,c_{2,2} -$$

$$12\,646\,416\,T^{14}\,c_{2,2} + 7\,205\,208\,T^{15}\,c_{2,2} - 7\,205\,208\,T^{17}\,c_{2,2} + 12\,646\,416\,T^{18}\,c_{2,2} - 15\,241\,728\,T^{19}\,c_{2,2} +$$

$$14\,922\,472\,T^{20}\,c_{2,2} - 12\,483\,640\,T^{21}\,c_{2,2} + 9\,103\,728\,T^{22}\,c_{2,2} - 5\,831\,784\,T^{23}\,c_{2,2} + 3\,284\,656\,T^{24}\,c_{2,2} -$$

$$1\,620\,216\,T^{25}\,c_{2,2} + 693\,960\,T^{26}\,c_{2,2} - 254\,496\,T^{27}\,c_{2,2} + 78\,192\,T^{28}\,c_{2,2} - 19\,448\,T^{29}\,c_{2,2} +$$

$$3\,696\,T^{30}\,c_{2,2} - 480\,T^{31}\,c_{2,2} + 32\,T^{32}\,c_{2,2} - 32\,c_{2,5} + 480\,T\,c_{2,5} - 3\,696\,T^2\,c_{2,5} + 19\,448\,T^3\,c_{2,5} -$$

$$78\,192\,T^4\,c_{2,5} + 254\,496\,T^5\,c_{2,5} - 693\,960\,T^6\,c_{2,5} + 1\,620\,216\,T^7\,c_{2,5} - 3\,284\,656\,T^8\,c_{2,5} +$$

$$5\,831\,784\,T^9\,c_{2,5} - 9\,103\,728\,T^{10}\,c_{2,5} + 12\,483\,640\,T^{11}\,c_{2,5} - 14\,922\,472\,T^{12}\,c_{2,5} + 15\,241\,728\,T^{13}\,c_{2,5} -$$

$$12\,646\,416\,T^{14}\,c_{2,5} + 7\,205\,208\,T^{15}\,c_{2,5} - 7\,205\,208\,T^{17}\,c_{2,5} + 12\,646\,416\,T^{18}\,c_{2,5} - 15\,241\,728\,T^{19}\,c_{2,5} +$$

$$14\,922\,472\,T^{20}\,c_{2,5} - 12\,483\,640\,T^{21}\,c_{2,5} + 9\,103\,728\,T^{22}\,c_{2,5} - 5\,831\,784\,T^{23}\,c_{2,5} + 3\,284\,656\,T^{24}\,c_{2,5} -$$

$$1\,620\,216\,T^{25}\,c_{2,5} + 693\,960\,T^{26}\,c_{2,5} - 254\,496\,T^{27}\,c_{2,5} + 78\,192\,T^{28}\,c_{2,5} - 19\,448\,T^{29}\,c_{2,5} +$$

$$3\,696\,T^{30}\,c_{2,5} - 480\,T^{31}\,c_{2,5} + 32\,T^{32}\,c_{2,5} - 24\,c_{2,10} + 352\,T\,c_{2,10} - 2\,648\,T^2\,c_{2,10} + 13\,592\,T^3\,c_{2,10} -$$

$$53\,208\,T^4\,c_{2,10} + 168\,192\,T^5\,c_{2,10} - 443\,688\,T^6\,c_{2,10} + 995\,864\,T^7\,c_{2,10} - 1\,920\,832\,T^8\,c_{2,10} +$$

$$3\,187\,896\,T^9\,c_{2,10} - 4\,506\,304\,T^{10}\,c_{2,10} + 5\,249\,976\,T^{11}\,c_{2,10} - 4\,549\,000\,T^{12}\,c_{2,10} + 1\,601\,312\,T^{13}\,c_{2,10} +$$

$$\begin{aligned} & 3\,879\,856\,T^{14}\,c_{2,10} - 11\,310\,184\,T^{15}\,c_{2,10} + 19\,225\,392\,T^{16}\,c_{2,10} - 25\,720\,600\,T^{17}\,c_{2,10} + \\ & 29\,172\,688\,T^{18}\,c_{2,10} - 28\,882\,144\,T^{19}\,c_{2,10} + 25\,295\,944\,T^{20}\,c_{2,10} - 19\,717\,304\,T^{21}\,c_{2,10} + \\ & 13\,701\,152\,T^{22}\,c_{2,10} - 8\,475\,672\,T^{23}\,c_{2,10} + 4\,648\,480\,T^{24}\,c_{2,10} - 2\,244\,568\,T^{25}\,c_{2,10} + 944\,232\,T^{26}\,c_{2,10} - \\ & 340\,800\,T^{27}\,c_{2,10} + 103\,176\,T^{28}\,c_{2,10} - 25\,304\,T^{29}\,c_{2,10} + 4\,744\,T^{30}\,c_{2,10} - 608\,T^{31}\,c_{2,10} + 40\,T^{32}\,c_{2,10} \Big) / \\ & \left(8 - 128\,T + 1056\,T^2 - 5984\,T^3 + 26\,064\,T^4 - 92\,544\,T^5 + 277\,584\,T^6 - 720\,096\,T^7 + 1\,642\,328\,T^8 - \right. \\ & 3\,332\,448\,T^9 + 6\,069\,152\,T^{10} - 9\,986\,912\,T^{11} + 14\,922\,472\,T^{12} - 20\,322\,304\,T^{13} + 25\,292\,832\,T^{14} - \\ & 28\,820\,832\,T^{15} + 30\,099\,512\,T^{16} - 28\,820\,832\,T^{17} + 25\,292\,832\,T^{18} - 20\,322\,304\,T^{19} + \\ & 14\,922\,472\,T^{20} - 9\,986\,912\,T^{21} + 6\,069\,152\,T^{22} - 3\,332\,448\,T^{23} + 1\,642\,328\,T^{24} - 720\,096\,T^{25} + \\ & \left. 277\,584\,T^{26} - 92\,544\,T^{27} + 26\,064\,T^{28} - 5984\,T^{29} + 1056\,T^{30} - 128\,T^{31} + 8\,T^{32} \right) \Big] \Big\} \end{aligned}$$

In[]:= {Z1[[3, 2]] == Z2[[3, 2]], Z1[[3, 3]] - Z2[[3, 3]]} // Simplify

$$\text{Out[]:= } \left\{ \text{True, } -\frac{(-1+T)^4 T^3 (1-3T+2T^2+5T^3-12T^4+18T^5-12T^6+5T^7+2T^8-3T^9+T^{10})}{(1-T+T^2)(1-3T+5T^2-7T^3+5T^4-3T^5+T^6)^3} \right\}$$

In[]:= {C1, C1_bar, CF[CF[Rp1,2 /. T -> T^-1][[3, 3]] - CF[Rp1_bar,2][[3, 3]]] /.
 {c2,2 -> 0, c2,5 -> 0, c2,10 -> 1/8, c2,11 -> -1/8, c2,26 -> 1/8}}

$$\begin{aligned} \text{Out[]:= } & \left\{ \mathbb{E}_{\{\} \rightarrow \{1\}} \left[\sqrt{T}, 0, \in \text{Series} \left[0, \frac{p_1 x_1}{2}, -\frac{1}{8} p_1 x_1 \right] \right], \right. \\ & \left. \mathbb{E}_{\{\} \rightarrow \{1\}} \left[\frac{1}{\sqrt{T}}, 0, \in \text{Series} \left[0, -\frac{1}{2} p_1 x_1, -\frac{1}{8} p_1 x_1 \right] \right], 0 \right\} \end{aligned}$$

In[]:= {c2,2 = 0, c2,5 = 0, c2,10 = 1/8, c2,11 = -1/8, c2,26 = 1/8};
 {R1,2, R1_bar,2, R1,2 /. T -> T^-1, C1, C1_bar} // CF // Column

$$\begin{aligned} & \mathbb{E}_{\{\} \rightarrow \{1,2\}} \left[\sqrt{T}, (-1+T) p_1 x_2 + (1-T) p_2 x_2, \right. \\ & \in \text{Series} \left[0, \frac{1}{4} (-1+T) p_1^2 x_1 x_2 - \frac{1}{2} T p_1 p_2 x_1 x_2 + \frac{1}{4} (T-T^2) p_1^2 x_2^2 + \frac{1}{2} T^2 p_1 p_2 x_2^2, \right. \\ & \frac{1}{16} (3-3T) p_1^2 x_1 x_2 + \frac{1}{8} T p_1 p_2 x_1 x_2 + \frac{1}{6} (-1+T) p_1^3 x_1^2 x_2 - \frac{1}{8} T p_1^2 p_2 x_1^2 x_2 + \\ & \frac{1}{16} (-T+T^2) p_1^2 x_2^2 - \frac{1}{8} T p_1 p_2 x_2^2 + \frac{1}{24} (-4+11T-7T^2) p_1^3 x_1 x_2^2 + \frac{1}{8} (-T+4T^2) p_1^2 p_2 x_1 x_2^2 - \\ & \left. \frac{1}{8} T^2 p_1 p_2^2 x_1 x_2^2 + \frac{1}{24} (T-4T^2+3T^3) p_1^3 x_2^3 + \frac{1}{8} (T-T^2-2T^3) p_1^2 p_2 x_2^3 + \frac{1}{8} T^2 p_1 p_2^2 x_2^3 \right] \Big] \\ & \mathbb{E}_{\{\} \rightarrow \{1,2\}} \left[\frac{1}{\sqrt{T}}, \frac{(1-T) p_1 x_2}{T} + \frac{(-1+T) p_2 x_2}{T}, \in \text{Series} \left[0, \frac{(-1+T) p_1^2 x_1 x_2}{4T} + \frac{p_1 p_2 x_1 x_2}{2T} + \frac{(1-T) p_1^2 x_2^2}{4T^2} - \frac{p_1 p_2 x_2^2}{2T^2}, \right. \right. \\ & \left. \frac{(-3+3T) p_1^2 x_1 x_2}{16T} + \frac{p_1 p_2 x_1 x_2}{8T} + \frac{(1-T) p_1^3 x_1^2 x_2}{6T} - \frac{p_1^2 p_2 x_1^2 x_2}{8T} + \frac{(1-T) p_1^2 x_2^2}{16T^2} - \frac{p_1 p_2 x_2^2}{8T} + \right. \\ & \left. \frac{(-7+11T-4T^2) p_1^3 x_1 x_2^2}{24T^2} + \frac{(4-T) p_1^2 p_2 x_1 x_2^2}{8T^2} - \frac{p_1 p_2^2 x_1 x_2^2}{8T^2} + \frac{(3-4T+T^2) p_1^3 x_2^3}{24T^3} + \frac{(-2-T+T^2) p_1^2 p_2 x_2^3}{8T^3} + \frac{p_1 p_2^2 x_2^3}{8T^2} \right] \Big] \\ & \mathbb{E}_{\{\} \rightarrow \{1,2\}} \left[\sqrt{\frac{1}{T}}, \frac{(1-T) p_1 x_2}{T} + \frac{(-1+T) p_2 x_2}{T}, \in \text{Series} \left[0, \frac{(1-T) p_1^2 x_1 x_2}{4T} - \frac{p_1 p_2 x_1 x_2}{2T} + \frac{(-1+T) p_1^2 x_2^2}{4T^2} + \frac{p_1 p_2 x_2^2}{2T^2}, \right. \right. \\ & \left. \frac{(-3+3T) p_1^2 x_1 x_2}{16T} + \frac{p_1 p_2 x_1 x_2}{8T} + \frac{(1-T) p_1^3 x_1^2 x_2}{6T} - \frac{p_1^2 p_2 x_1^2 x_2}{8T} + \frac{(1-T) p_1^2 x_2^2}{16T^2} - \frac{p_1 p_2 x_2^2}{8T} + \right. \\ & \left. \frac{(-7+11T-4T^2) p_1^3 x_1 x_2^2}{24T^2} + \frac{(4-T) p_1^2 p_2 x_1 x_2^2}{8T^2} - \frac{p_1 p_2^2 x_1 x_2^2}{8T^2} + \frac{(3-4T+T^2) p_1^3 x_2^3}{24T^3} + \frac{(-2-T+T^2) p_1^2 p_2 x_2^3}{8T^3} + \frac{p_1 p_2^2 x_2^3}{8T^2} \right] \Big] \\ & \mathbb{E}_{\{\} \rightarrow \{1\}} \left[\sqrt{T}, 0, \in \text{Series} \left[0, \frac{p_1 x_1}{2}, -\frac{1}{8} p_1 x_1 \right] \right] \\ & \mathbb{E}_{\{\} \rightarrow \{1\}} \left[\frac{1}{\sqrt{T}}, 0, \in \text{Series} \left[0, -\frac{1}{2} p_1 x_1, -\frac{1}{8} p_1 x_1 \right] \right] \end{aligned}$$

In[*]:= **CF**[**R**_{1,2} /. **T** → **T**⁻¹][**3**, **3**] - **CF**[**R**_{1,2}][**3**, **3**]

Out[*]:= 0

In[*]:= **Factor** /@ **R**_{1,2}[**3**]

Out[*]:= **Series**[0, $\frac{1}{4} p_1 (-p_1 + T p_1 + 2 p_2) x_2 (-x_1 + T x_2)$,
 $-\frac{1}{48} p_1 x_2 (-3 p_1 x_1 + 3 T p_1 x_1 - 6 p_2 x_1 + 2 p_1^2 x_1^2 - 2 T p_1^2 x_1^2 + 6 p_1 p_2 x_1^2 - 6 p_1 x_2 + 9 T p_1 x_2 -$
 $3 T^2 p_1 x_2 + 6 p_2 x_2 + 8 p_1^2 x_1 x_2 - 4 T p_1^2 x_1 x_2 - 4 T^2 p_1^2 x_1 x_2 - 6 p_1 p_2 x_1 x_2 - 12 T p_1 p_2 x_1 x_2 +$
 $6 p_2^2 x_1 x_2 - 2 T p_1^2 x_2^2 - 4 T^2 p_1^2 x_2^2 + 6 T^3 p_1^2 x_2^2 + 6 p_1 p_2 x_2^2 - 6 T p_1 p_2 x_2^2 + 12 T^2 p_1 p_2 x_2^2 - 6 p_2^2 x_2^2)$]

In[*]:= **Simplify** /@ **R**_{1,2}[**3**]

Out[*]:= **Series**[0, $\frac{1}{4} p_1 ((-1 + T) p_1 + 2 p_2) x_2 (-x_1 + T x_2)$,
 $-\frac{1}{48} p_1 x_2 (6 p_2 (x_1 - x_2) (-1 + p_2 x_2) - 2 (-1 + T) p_1^2 (x_1^2 + 2 (2 + T) x_1 x_2 - T (1 + 3 T) x_2^2) +$
 $3 p_1 (2 p_2 x_1^2 + x_1 (-1 + T - 2 (1 + 2 T) p_2 x_2) + x_2 (-2 + 3 T - T^2 + 2 (1 - T + 2 T^2) p_2 x_2)))]$

In[*]:= **RMoves**

Out[*]:= {True, True, True, True, True, True, True, True}

Solving for R, C, \$k = 3

In[*]:= **\$k = 3;**

{R_{1,2}, C₁}

Out[*]:= $\{E_{\{1,2\}}[\sqrt{T}, (-1 + T) p_1 x_2 + (1 - T) p_2 x_2,$
 $\in \text{Series}[0, \frac{1}{4} (-1 + T) p_1^2 x_1 x_2 - \frac{1}{2} T p_1 p_2 x_1 x_2 + \frac{1}{4} (T - T^2) p_1^2 x_2^2 + \frac{1}{2} T^2 p_1 p_2 x_2^2,$
 $\frac{1}{16} (3 - 3 T) p_1^2 x_1 x_2 + \frac{1}{8} T p_1 p_2 x_1 x_2 + \frac{1}{6} (-1 + T) p_1^3 x_1^2 x_2 - \frac{1}{8} T p_1^2 p_2 x_1^2 x_2 +$
 $\frac{1}{16} (-T + T^2) p_1^2 x_2^2 - \frac{1}{8} T p_1 p_2 x_2^2 + \frac{1}{24} (-4 + 11 T - 7 T^2) p_1^3 x_1 x_2^2 + \frac{1}{8} (-T + 4 T^2) p_1^2 p_2 x_1 x_2^2 -$
 $\frac{1}{8} T^2 p_1 p_2^2 x_1 x_2^2 + \frac{1}{24} (T - 4 T^2 + 3 T^3) p_1^3 x_2^3 + \frac{1}{8} (T - T^2 - 2 T^3) p_1^2 p_2 x_2^3 + \frac{1}{8} T^2 p_1 p_2^2 x_2^3,$
 $c_{3,1} + T p_2 x_1 c_{3,4} + p_1 x_1 (c_{3,2} + c_{3,4} - T c_{3,4}) + T p_2 x_2 c_{3,5} + p_1 x_2 (c_{3,3} + c_{3,5} - T c_{3,5}) + T^2 p_2^2 x_1^2 c_{3,12} +$
 $p_1 p_2 x_1^2 (T c_{3,9} + 2 T c_{3,12} - 2 T^2 c_{3,12}) + p_1^2 x_1^2 (c_{3,6} + c_{3,9} - T c_{3,9} + c_{3,12} - 2 T c_{3,12} + T^2 c_{3,12}) +$
 $T^2 p_2^2 x_1 x_2 c_{3,13} + p_1 p_2 x_1 x_2 (T c_{3,10} + 2 T c_{3,13} - 2 T^2 c_{3,13}) +$
 $p_1^2 x_1 x_2 (c_{3,7} + c_{3,10} - T c_{3,10} + c_{3,13} - 2 T c_{3,13} + T^2 c_{3,13}) + T^2 p_2^2 x_2^2 c_{3,14} +$
 $p_1 p_2 x_2^2 (T c_{3,11} + 2 T c_{3,14} - 2 T^2 c_{3,14}) + p_1^2 x_2^2 (c_{3,8} + c_{3,11} - T c_{3,11} + c_{3,14} - 2 T c_{3,14} + T^2 c_{3,14}) +$
 $T^3 p_2^3 x_1^3 c_{3,27} + p_1 p_2^2 x_1^3 (T^2 c_{3,23} + 3 T^2 c_{3,27} - 3 T^3 c_{3,27}) +$
 $p_1^3 x_1^3 (c_{3,15} + c_{3,19} - T c_{3,19} + c_{3,23} - 2 T c_{3,23} + T^2 c_{3,23} + c_{3,27} - 3 T c_{3,27} + 3 T^2 c_{3,27} - T^3 c_{3,27}) +$
 $p_1^2 p_2 x_1^3 (T c_{3,19} + 2 T c_{3,23} - 2 T^2 c_{3,23} + 3 T c_{3,27} - 6 T^2 c_{3,27} + 3 T^3 c_{3,27}) +$

$$\begin{aligned}
 & T^3 p_2^3 x_1^2 x_2 c_{3,28} + p_1 p_2^2 x_1^2 x_2 (T^2 c_{3,24} + 3 T^2 c_{3,28} - 3 T^3 c_{3,28}) + \\
 & p_1^3 x_1^2 x_2 (c_{3,16} + c_{3,20} - T c_{3,20} + c_{3,24} - 2 T c_{3,24} + T^2 c_{3,24} + c_{3,28} - 3 T c_{3,28} + 3 T^2 c_{3,28} - T^3 c_{3,28}) + \\
 & p_1^2 p_2 x_1^2 x_2 (T c_{3,20} + 2 T c_{3,24} - 2 T^2 c_{3,24} + 3 T c_{3,28} - 6 T^2 c_{3,28} + 3 T^3 c_{3,28}) + \\
 & T^3 p_2^3 x_1 x_2^2 c_{3,29} + p_1 p_2^2 x_1 x_2^2 (T^2 c_{3,25} + 3 T^2 c_{3,29} - 3 T^3 c_{3,29}) + \\
 & p_1^3 x_1 x_2^2 (c_{3,17} + c_{3,21} - T c_{3,21} + c_{3,25} - 2 T c_{3,25} + T^2 c_{3,25} + c_{3,29} - 3 T c_{3,29} + 3 T^2 c_{3,29} - T^3 c_{3,29}) + \\
 & p_1^2 p_2 x_1 x_2^2 (T c_{3,21} + 2 T c_{3,25} - 2 T^2 c_{3,25} + 3 T c_{3,29} - 6 T^2 c_{3,29} + 3 T^3 c_{3,29}) + \\
 & T^3 p_2^3 x_2^3 c_{3,30} + p_1 p_2^2 x_2^3 (T^2 c_{3,26} + 3 T^2 c_{3,30} - 3 T^3 c_{3,30}) + \\
 & p_1^3 x_2^3 (c_{3,18} + c_{3,22} - T c_{3,22} + c_{3,26} - 2 T c_{3,26} + T^2 c_{3,26} + c_{3,30} - 3 T c_{3,30} + 3 T^2 c_{3,30} - T^3 c_{3,30}) + \\
 & p_1^2 p_2 x_2^3 (T c_{3,22} + 2 T c_{3,26} - 2 T^2 c_{3,26} + 3 T c_{3,30} - 6 T^2 c_{3,30} + 3 T^3 c_{3,30}) + T^4 p_2^4 x_1^4 c_{3,51} + \\
 & p_1 p_2^3 x_1^4 (T^3 c_{3,46} + 4 T^3 c_{3,51} - 4 T^4 c_{3,51}) + p_1^3 p_2 x_1^4 (T c_{3,36} + 2 T c_{3,41} - 2 T^2 c_{3,41} + \\
 & \quad 3 T c_{3,46} - 6 T^2 c_{3,46} + 3 T^3 c_{3,46} + 4 T c_{3,51} - 12 T^2 c_{3,51} + 12 T^3 c_{3,51} - 4 T^4 c_{3,51}) + \\
 & p_1^4 x_1^4 (c_{3,31} + c_{3,36} - T c_{3,36} + c_{3,41} - 2 T c_{3,41} + T^2 c_{3,41} + c_{3,46} - 3 T c_{3,46} + 3 T^2 c_{3,46} - \\
 & \quad T^3 c_{3,46} + c_{3,51} - 4 T c_{3,51} + 6 T^2 c_{3,51} - 4 T^3 c_{3,51} + T^4 c_{3,51}) + \\
 & p_1^2 p_2^2 x_1^4 (T^2 c_{3,41} + 3 T^2 c_{3,46} - 3 T^3 c_{3,46} + 6 T^2 c_{3,51} - 12 T^3 c_{3,51} + 6 T^4 c_{3,51}) + T^4 p_2^4 x_1^3 x_2 c_{3,52} + \\
 & p_1 p_2^3 x_1^3 x_2 (T^3 c_{3,47} + 4 T^3 c_{3,52} - 4 T^4 c_{3,52}) + p_1^3 p_2 x_1^3 x_2 (T c_{3,37} + 2 T c_{3,42} - 2 T^2 c_{3,42} + \\
 & \quad 3 T c_{3,47} - 6 T^2 c_{3,47} + 3 T^3 c_{3,47} + 4 T c_{3,52} - 12 T^2 c_{3,52} + 12 T^3 c_{3,52} - 4 T^4 c_{3,52}) + \\
 & p_1^4 x_1^3 x_2 (c_{3,32} + c_{3,37} - T c_{3,37} + c_{3,42} - 2 T c_{3,42} + T^2 c_{3,42} + c_{3,47} - 3 T c_{3,47} + \\
 & \quad 3 T^2 c_{3,47} - T^3 c_{3,47} + c_{3,52} - 4 T c_{3,52} + 6 T^2 c_{3,52} - 4 T^3 c_{3,52} + T^4 c_{3,52}) + \\
 & p_1^2 p_2^2 x_1^3 x_2 (T^2 c_{3,42} + 3 T^2 c_{3,47} - 3 T^3 c_{3,47} + 6 T^2 c_{3,52} - 12 T^3 c_{3,52} + 6 T^4 c_{3,52}) + \\
 & T^4 p_2^4 x_1^2 x_2^2 c_{3,53} + p_1 p_2^3 x_1^2 x_2^2 (T^3 c_{3,48} + 4 T^3 c_{3,53} - 4 T^4 c_{3,53}) + \\
 & p_1^3 p_2 x_1^2 x_2^2 (T c_{3,38} + 2 T c_{3,43} - 2 T^2 c_{3,43} + 3 T c_{3,48} - 6 T^2 c_{3,48} + 3 T^3 c_{3,48} + 4 T c_{3,53} - 12 T^2 c_{3,53} + \\
 & \quad 12 T^3 c_{3,53} - 4 T^4 c_{3,53}) + p_1^4 x_1^2 x_2^2 (c_{3,33} + c_{3,38} - T c_{3,38} + c_{3,43} - 2 T c_{3,43} + T^2 c_{3,43} + \\
 & \quad c_{3,48} - 3 T c_{3,48} + 3 T^2 c_{3,48} - T^3 c_{3,48} + c_{3,53} - 4 T c_{3,53} + 6 T^2 c_{3,53} - 4 T^3 c_{3,53} + T^4 c_{3,53}) + \\
 & p_1^2 p_2^2 x_1^2 x_2^2 (T^2 c_{3,43} + 3 T^2 c_{3,48} - 3 T^3 c_{3,48} + 6 T^2 c_{3,53} - 12 T^3 c_{3,53} + 6 T^4 c_{3,53}) + \\
 & T^4 p_2^4 x_1 x_2^3 c_{3,54} + p_1 p_2^3 x_1 x_2^3 (T^3 c_{3,49} + 4 T^3 c_{3,54} - 4 T^4 c_{3,54}) + \\
 & p_1^3 p_2 x_1 x_2^3 (T c_{3,39} + 2 T c_{3,44} - 2 T^2 c_{3,44} + 3 T c_{3,49} - 6 T^2 c_{3,49} + 3 T^3 c_{3,49} + 4 T c_{3,54} - 12 T^2 c_{3,54} + \\
 & \quad 12 T^3 c_{3,54} - 4 T^4 c_{3,54}) + p_1^4 x_1 x_2^3 (c_{3,34} + c_{3,39} - T c_{3,39} + c_{3,44} - 2 T c_{3,44} + T^2 c_{3,44} + \\
 & \quad c_{3,49} - 3 T c_{3,49} + 3 T^2 c_{3,49} - T^3 c_{3,49} + c_{3,54} - 4 T c_{3,54} + 6 T^2 c_{3,54} - 4 T^3 c_{3,54} + T^4 c_{3,54}) + \\
 & p_1^2 p_2^2 x_1 x_2^3 (T^2 c_{3,44} + 3 T^2 c_{3,49} - 3 T^3 c_{3,49} + 6 T^2 c_{3,54} - 12 T^3 c_{3,54} + 6 T^4 c_{3,54}) + \\
 & T^4 p_2^4 x_2^4 c_{3,55} + p_1 p_2^3 x_2^4 (T^3 c_{3,50} + 4 T^3 c_{3,55} - 4 T^4 c_{3,55}) + p_1^3 p_2 x_2^4 (T c_{3,40} + 2 T c_{3,45} - 2 T^2 c_{3,45} + \\
 & \quad 3 T c_{3,50} - 6 T^2 c_{3,50} + 3 T^3 c_{3,50} + 4 T c_{3,55} - 12 T^2 c_{3,55} + 12 T^3 c_{3,55} - 4 T^4 c_{3,55}) + \\
 & p_1^4 x_2^4 (c_{3,35} + c_{3,40} - T c_{3,40} + c_{3,45} - 2 T c_{3,45} + T^2 c_{3,45} + c_{3,50} - 3 T c_{3,50} + 3 T^2 c_{3,50} - \\
 & \quad T^3 c_{3,50} + c_{3,55} - 4 T c_{3,55} + 6 T^2 c_{3,55} - 4 T^3 c_{3,55} + T^4 c_{3,55}) + \\
 & p_1^2 p_2^2 x_2^4 (T^2 c_{3,45} + 3 T^2 c_{3,50} - 3 T^3 c_{3,50} + 6 T^2 c_{3,55} - 12 T^3 c_{3,55} + 6 T^4 c_{3,55}) \Big], \\
 & E_{\{\} \rightarrow \{1\}} \left[\sqrt{T}, \theta, \in \text{Series} \left[\theta, \frac{p_1 x_1}{2}, -\frac{1}{8} p_1 x_1, e_{3,1} + p_1 x_1 e_{3,2} + p_1^2 x_1^2 e_{3,3} + p_1^3 x_1^3 e_{3,4} + p_1^4 x_1^4 e_{3,5} \right] \right] \Big\}
 \end{aligned}$$

In[]:= unknowns = Cases [{R_{1,2}, R̄_{1,2}, C₁, C̄₁}, (c | d | e | f)_{\$k,_, ∞}] // Union

Out[*]= {C3,1, C3,2, C3,3, C3,4, C3,5, C3,6, C3,7, C3,8, C3,9, C3,10, C3,11, C3,12, C3,13, C3,14, C3,15, C3,16, C3,17, C3,18, C3,19, C3,20, C3,21, C3,22, C3,23, C3,24, C3,25, C3,26, C3,27, C3,28, C3,29, C3,30, C3,31, C3,32, C3,33, C3,34, C3,35, C3,36, C3,37, C3,38, C3,39, C3,40, C3,41, C3,42, C3,43, C3,44, C3,45, C3,46, C3,47, C3,48, C3,49, C3,50, C3,51, C3,52, C3,53, C3,54, C3,55, d3,1, d3,2, d3,3, d3,4, d3,5, d3,6, d3,7, d3,8, d3,9, d3,10, d3,11, d3,12, d3,13, d3,14, d3,15, d3,16, d3,17, d3,18, d3,19, d3,20, d3,21, d3,22, d3,23, d3,24, d3,25, d3,26, d3,27, d3,28, d3,29, d3,30, d3,31, d3,32, d3,33, d3,34, d3,35, d3,36, d3,37, d3,38, d3,39, d3,40, d3,41, d3,42, d3,43, d3,44, d3,45, d3,46, d3,47, d3,48, d3,49, d3,50, d3,51, d3,52, d3,53, d3,54, d3,55, e3,1, e3,2, e3,3, e3,4, e3,5, f3,1, f3,2, f3,3, f3,4, f3,5}

In[*]:= Short[errors = CCF[@Cases[RMoves, a_ == b_ => a - b], 25]

Out[*]//Short=

$$\left\{ \frac{1}{192} \left(12 T p_1 p_2 x_2 x_3 - 12 T^3 p_1 p_2 x_2 x_3 + 80 T p_1^3 x_1 x_2 x_3 - 160 T^2 p_1^3 x_1 x_2 x_3 + 80 T^3 p_1^3 x_1 x_2 x_3 + \right. \right.$$

$$12 T p_1^2 p_2 x_1 x_2 x_3 + 72 T^2 p_1^2 p_2 x_1 x_2 x_3 - 84 T^3 p_1^2 p_2 x_1 x_2 x_3 - 24 T^2 p_1 p_2^2 x_1 x_2 x_3 +$$

$$24 T^3 p_1 p_2^2 x_1 x_2 x_3 + 24 T^2 p_1^2 p_3 x_1 x_2 x_3 - 24 T^3 p_1^2 p_3 x_1 x_2 x_3 + 37 p_1^4 x_1^2 x_2 x_3 - 149 T p_1^4 x_1^2 x_2 x_3 +$$

$$187 T^2 p_1^4 x_1^2 x_2 x_3 - 75 T^3 p_1^4 x_1^2 x_2 x_3 + 76 T p_1^3 p_2 x_1^2 x_2 x_3 - 220 T^2 p_1^3 p_2 x_1^2 x_2 x_3 + 144 T^3 p_1^3 p_2 x_1^2 x_2 x_3 +$$

$$48 T^2 p_1^2 p_2^2 x_1^2 x_2 x_3 - 48 T^3 p_1^2 p_2^2 x_1^2 x_2 x_3 - 12 T^2 p_1^3 p_3 x_1^2 x_2 x_3 + 12 T^3 p_1^3 p_3 x_1^2 x_2 x_3 +$$

$$4 T^2 p_1^3 x_2^2 x_3 - 8 T^3 p_1^3 x_2^2 x_3 + 4 T^4 p_1^3 x_2^2 x_3 - 54 T p_1^2 p_2 x_2^2 x_3 - 12 T^2 p_1^2 p_2 x_2^2 x_3 + 138 T^3 p_1^2 p_2 x_2^2 x_3 -$$

$$72 T^4 p_1^2 p_2 x_2^2 x_3 - 66 T^2 p_1 p_2^2 x_2^2 x_3 + 36 T^3 p_1 p_2^2 x_2^2 x_3 + 30 T^4 p_1 p_2^2 x_2^2 x_3 - 24 T^3 p_1 p_2 p_3 x_2^2 x_3 +$$

$$24 T^4 p_1 p_2 p_3 x_2^2 x_3 - 77 T p_1^4 x_1 x_2^2 x_3 + \ll 8170 \gg + 16128 T^3 p_1^3 p_2 x_3^4 c_{3,55} - 26880 T^4 p_1^3 p_2 x_3^4 c_{3,55} +$$

$$26880 T^5 p_1^3 p_2 x_3^4 c_{3,55} - 16128 T^6 p_1^3 p_2 x_3^4 c_{3,55} + 5376 T^7 p_1^3 p_2 x_3^4 c_{3,55} - 768 T^8 p_1^3 p_2 x_3^4 c_{3,55} +$$

$$1152 T^2 p_1^2 p_2^2 x_3^4 c_{3,55} - 6912 T^3 p_1^2 p_2^2 x_3^4 c_{3,55} + 17280 T^4 p_1^2 p_2^2 x_3^4 c_{3,55} - 23040 T^5 p_1^2 p_2^2 x_3^4 c_{3,55} +$$

$$17280 T^6 p_1^2 p_2^2 x_3^4 c_{3,55} - 6912 T^7 p_1^2 p_2^2 x_3^4 c_{3,55} + 1152 T^8 p_1^2 p_2^2 x_3^4 c_{3,55} + 768 T^3 p_1 p_2^3 x_3^4 c_{3,55} -$$

$$3840 T^4 p_1 p_2^3 x_3^4 c_{3,55} + 7680 T^5 p_1 p_2^3 x_3^4 c_{3,55} - 7680 T^6 p_1 p_2^3 x_3^4 c_{3,55} + 3840 T^7 p_1 p_2^3 x_3^4 c_{3,55} -$$

$$768 T^8 p_1 p_2^3 x_3^4 c_{3,55} + 768 T^5 p_1^3 p_3 x_3^4 c_{3,55} - 2304 T^6 p_1^3 p_3 x_3^4 c_{3,55} + 2304 T^7 p_1^3 p_3 x_3^4 c_{3,55} -$$

$$768 T^8 p_1^3 p_3 x_3^4 c_{3,55} - 768 T^5 p_2^3 p_3 x_3^4 c_{3,55} + 2304 T^6 p_2^3 p_3 x_3^4 c_{3,55} - 2304 T^7 p_2^3 p_3 x_3^4 c_{3,55} +$$

$$768 T^8 p_2^3 p_3 x_3^4 c_{3,55} + 1152 T^6 p_1^2 p_3^2 x_3^4 c_{3,55} - 2304 T^7 p_1^2 p_3^2 x_3^4 c_{3,55} + 1152 T^8 p_1^2 p_3^2 x_3^4 c_{3,55} -$$

$$1152 T^6 p_2^2 p_3^2 x_3^4 c_{3,55} + 2304 T^7 p_2^2 p_3^2 x_3^4 c_{3,55} - 1152 T^8 p_2^2 p_3^2 x_3^4 c_{3,55} + 768 T^7 p_1 p_3^3 x_3^4 c_{3,55} -$$

$$768 T^8 p_1 p_3^3 x_3^4 c_{3,55} - 768 T^7 p_2 p_3^3 x_3^4 c_{3,55} + 768 T^8 p_2 p_3^3 x_3^4 c_{3,55} \Big), \ll 7 \gg, \frac{-12 T + \ll 402 \gg + \ll 1 \gg}{96 T^4} \Big\}$$

In[*]:= Short[# , 10] &[eqns =

Thread[0 == Union@@(CoefficientRules[# , {x1, x2, x3, p1, p2, p3}][[; , 2] & /@ errors)]]

Out[*]//Short=

$$\left\{ \theta == T c_{3,4} - T^2 c_{3,4}, \theta == -T c_{3,4} + T^2 c_{3,4} \ll 1 \gg \ll 1 \gg, \ll 490 \gg, \theta == \ll 1 \gg, \right.$$

$$\theta == \frac{3}{8} + \frac{23 T}{16} - T^2 - \frac{15 T^3}{16} + d_{3,6} + T d_{3,7} + T^2 d_{3,8} + d_{3,9} + T d_{3,10} + T^2 d_{3,11} + d_{3,12} + T d_{3,13} + T^2 d_{3,14} -$$

$$3 T d_{3,16} - 6 T^2 d_{3,17} - 9 T^3 d_{3,18} + d_{3,20} - 3 T d_{3,20} + 2 T d_{3,21} - 6 T^2 d_{3,21} + 3 T^2 d_{3,22} - 9 T^3 d_{3,22} + 2 d_{3,24} -$$

$$3 T d_{3,24} + 4 T d_{3,25} - 6 T^2 d_{3,25} + 6 T^2 d_{3,26} - 9 T^3 d_{3,26} + 3 d_{3,28} - 3 T d_{3,28} + 6 T d_{3,29} - 6 T^2 d_{3,29} +$$

$$\ll 18 \gg + 12 T^2 d_{3,43} + 6 T d_{3,44} - 36 T^2 d_{3,44} + 36 T^3 d_{3,44} + 12 T^2 d_{3,45} - 72 T^3 d_{3,45} + 72 T^4 d_{3,45} +$$

$$6 d_{3,48} - 18 T d_{3,48} + 12 T^2 d_{3,48} + 18 T d_{3,49} - 54 T^2 d_{3,49} + 36 T^3 d_{3,49} + 36 T^2 d_{3,50} - 108 T^3 d_{3,50} +$$

$$72 T^4 d_{3,50} + 12 d_{3,53} - 24 T d_{3,53} + 12 T^2 d_{3,53} + 36 T d_{3,54} - 72 T^2 d_{3,54} + 36 T^3 d_{3,54} + 72 T^2 d_{3,55} -$$

$$144 T^3 d_{3,55} + 72 T^4 d_{3,55} + T^2 f_{3,3} + 9 T^2 f_{3,4} - 9 T^3 f_{3,4} + 72 T^2 f_{3,5} - 144 T^3 f_{3,5} + 72 T^4 f_{3,5} \Big\}$$

In[*]:= {sol} = Solve[eqns, unknowns]

Solve: Equations may not give solutions for all "solve" variables.

$$\begin{aligned}
 \text{Out[*]} = & \left\{ \left\{ c_{3,1} \rightarrow -\frac{c_{3,2}}{2} - \frac{c_{3,5}}{2}, c_{3,3} \rightarrow -T c_{3,2} - c_{3,5}, c_{3,4} \rightarrow 0, c_{3,6} \rightarrow 0, \right. \right. \\
 & c_{3,7} \rightarrow \frac{1}{32} (-1 - T) - \frac{1}{2} (1 + T) c_{3,10} - c_{3,11}, c_{3,8} \rightarrow -\frac{1}{2} (T - T^2) c_{3,10} - (1 - T) c_{3,11}, \\
 & c_{3,9} \rightarrow 0, c_{3,12} \rightarrow 0, c_{3,13} \rightarrow 0, c_{3,14} \rightarrow 0, c_{3,15} \rightarrow 0, c_{3,16} \rightarrow \frac{1}{96} (5 + 7 T) - c_{3,26}, \\
 & c_{3,17} \rightarrow \frac{1}{48} (7 - 3 T - 4 T^2) - (1 - T) c_{3,26}, c_{3,18} \rightarrow \frac{1}{48} (-7 T + 8 T^2 - T^3), c_{3,19} \rightarrow 0, \\
 & c_{3,20} \rightarrow \frac{1}{16}, c_{3,21} \rightarrow \frac{1}{32} (-3 - 7 T), c_{3,22} \rightarrow \frac{1}{16} (2 T + T^2) - (1 - T) c_{3,26}, c_{3,23} \rightarrow 0, \\
 & c_{3,24} \rightarrow 0, c_{3,25} \rightarrow \frac{1}{16}, c_{3,27} \rightarrow 0, c_{3,28} \rightarrow 0, c_{3,29} \rightarrow 0, c_{3,30} \rightarrow 0, c_{3,31} \rightarrow 0, \\
 & c_{3,32} \rightarrow \frac{1}{192} (-3 - 5 T) - c_{3,50}, c_{3,33} \rightarrow \frac{1}{64} (-3 - 5 T + 8 T^2) + \frac{3}{2} (-1 + T) c_{3,50}, \\
 & c_{3,34} \rightarrow \frac{1}{192} (13 - 23 T + 43 T^2 - 33 T^3) - (1 - 2 T + T^2) c_{3,50}, \\
 & c_{3,35} \rightarrow \frac{1}{192} (-17 T + 36 T^2 - 39 T^3 + 20 T^4) - \frac{1}{2} (1 - 3 T + 3 T^2 - T^3) c_{3,50}, \\
 & c_{3,36} \rightarrow 0, c_{3,37} \rightarrow -\frac{1}{48}, c_{3,38} \rightarrow \frac{1}{48} (1 + 9 T), c_{3,39} \rightarrow \frac{1}{48} (-13 + 13 T - 12 T^2), \\
 & c_{3,40} \rightarrow \frac{1}{16} (5 T - 6 T^2 + 3 T^3) - (-1 + 2 T - T^2) c_{3,50}, c_{3,41} \rightarrow 0, c_{3,42} \rightarrow 0, c_{3,43} \rightarrow -\frac{1}{8}, \\
 & c_{3,44} \rightarrow \frac{1}{32} (6 + 5 T), c_{3,45} \rightarrow -\frac{7 T}{32} + \frac{3}{2} (-1 + T) c_{3,50}, c_{3,46} \rightarrow 0, c_{3,47} \rightarrow 0, c_{3,48} \rightarrow 0, c_{3,49} \rightarrow -\frac{1}{48}, \\
 & c_{3,51} \rightarrow 0, c_{3,52} \rightarrow 0, c_{3,53} \rightarrow 0, c_{3,54} \rightarrow 0, c_{3,55} \rightarrow 0, d_{3,1} \rightarrow \frac{c_{3,2}}{2} + \frac{c_{3,5}}{2}, d_{3,2} \rightarrow -c_{3,2}, \\
 & d_{3,3} \rightarrow \frac{c_{3,2}}{T} + c_{3,5}, d_{3,4} \rightarrow 0, d_{3,5} \rightarrow -c_{3,5}, d_{3,6} \rightarrow 0, d_{3,7} \rightarrow -\frac{1 - 3 T}{32 T} - \frac{(1 - 3 T) c_{3,10}}{2 T} + \frac{c_{3,11}}{T}, \\
 & d_{3,8} \rightarrow -\frac{-1 + 2 T - T^2}{16 T^2} - \frac{(-1 + 3 T - 2 T^2) c_{3,10}}{2 T^2} - \frac{(1 - T) c_{3,11}}{T^2}, d_{3,9} \rightarrow 0, d_{3,10} \rightarrow -c_{3,10}, \\
 & d_{3,11} \rightarrow -\frac{-1 + T}{16 T} - \frac{(-1 + T) c_{3,10}}{T} - \frac{c_{3,11}}{T}, d_{3,12} \rightarrow 0, d_{3,13} \rightarrow 0, d_{3,14} \rightarrow 0, \\
 & d_{3,15} \rightarrow 0, d_{3,16} \rightarrow -\frac{-5 + 17 T}{96 T} + \frac{c_{3,26}}{T}, d_{3,17} \rightarrow -\frac{2 - 15 T + 13 T^2}{48 T^2} - \frac{(1 - T) c_{3,26}}{T^2}, \\
 & d_{3,18} \rightarrow -\frac{-1 + 8 T - 7 T^2}{48 T^3}, d_{3,19} \rightarrow 0, d_{3,20} \rightarrow -\frac{1}{16}, d_{3,21} \rightarrow -\frac{-7 - 3 T}{32 T}, \\
 & d_{3,22} \rightarrow -\frac{3 - 2 T + 2 T^2}{16 T^2} - \frac{(1 - T) c_{3,26}}{T^2}, d_{3,23} \rightarrow 0, d_{3,24} \rightarrow 0, d_{3,25} \rightarrow -\frac{1}{16}, d_{3,26} \rightarrow -\frac{1 - T}{8 T} - \frac{c_{3,26}}{T}, \\
 & d_{3,27} \rightarrow 0, d_{3,28} \rightarrow 0, d_{3,29} \rightarrow 0, d_{3,30} \rightarrow 0, d_{3,31} \rightarrow 0, d_{3,32} \rightarrow -\frac{3 - 11 T}{192 T} + \frac{c_{3,50}}{T},
 \end{aligned}$$

$$\begin{aligned}
d_{3,33} &\rightarrow -\frac{4+3T-7T^2}{64T^2} + \frac{3(-1+T)c_{3,50}}{2T^2}, d_{3,34} \rightarrow -\frac{-25+19T+T^2+5T^3}{192T^3} - \frac{(-1+2T-T^2)c_{3,50}}{T^3}, \\
d_{3,35} &\rightarrow -\frac{16-23T+12T^2-T^3-4T^4}{192T^4} - \frac{(1-3T+3T^2-T^3)c_{3,50}}{2T^4}, d_{3,36} \rightarrow 0, d_{3,37} \rightarrow \frac{1}{48}, \\
d_{3,38} &\rightarrow -\frac{9+T}{48T}, d_{3,39} \rightarrow -\frac{-12+13T-13T^2}{48T^2}, d_{3,40} \rightarrow -\frac{7-12T+9T^2+2T^3}{48T^3} - \frac{(1-2T+T^2)c_{3,50}}{T^3}, \\
d_{3,41} &\rightarrow 0, d_{3,42} \rightarrow 0, d_{3,43} \rightarrow \frac{1}{8}, d_{3,44} \rightarrow -\frac{5+6T}{32T}, d_{3,45} \rightarrow -\frac{-2-3T-2T^2}{32T^2} + \frac{3(-1+T)c_{3,50}}{2T^2}, \\
d_{3,46} &\rightarrow 0, d_{3,47} \rightarrow 0, d_{3,48} \rightarrow 0, d_{3,49} \rightarrow \frac{1}{48}, d_{3,50} \rightarrow -\frac{-1+T}{24T} - \frac{c_{3,50}}{T}, d_{3,51} \rightarrow 0, \\
d_{3,52} &\rightarrow 0, d_{3,53} \rightarrow 0, d_{3,54} \rightarrow 0, d_{3,55} \rightarrow 0, e_{3,1} \rightarrow -\frac{c_{3,2}}{2} - \frac{c_{3,5}}{2}, e_{3,2} \rightarrow -c_{3,10}, e_{3,3} \rightarrow 0, \\
e_{3,4} &\rightarrow 0, e_{3,5} \rightarrow 0, f_{3,1} \rightarrow \frac{c_{3,2}}{2} + \frac{c_{3,5}}{2}, f_{3,2} \rightarrow c_{3,10}, f_{3,3} \rightarrow 0, f_{3,4} \rightarrow 0, f_{3,5} \rightarrow 0 \}}
\end{aligned}$$

In[*]:= sol /. (a_ -> b_) :-> (a = b)

$$\begin{aligned}
 \text{Out[*]} = & \left\{ -\frac{c_{3,2}}{2} - \frac{c_{3,5}}{2}, -T c_{3,2} - c_{3,5}, 0, 0, \frac{1}{32} (-1 - T) - \frac{1}{2} (1 + T) c_{3,10} - c_{3,11}, \right. \\
 & -\frac{1}{2} (T - T^2) c_{3,10} - (1 - T) c_{3,11}, 0, 0, 0, 0, 0, \frac{1}{96} (5 + 7 T) - c_{3,26}, \\
 & \frac{1}{48} (7 - 3 T - 4 T^2) - (1 - T) c_{3,26}, \frac{1}{48} (-7 T + 8 T^2 - T^3), 0, \frac{1}{16}, \frac{1}{32} (-3 - 7 T), \\
 & \frac{1}{16} (2 T + T^2) - (1 - T) c_{3,26}, 0, 0, \frac{1}{16}, 0, 0, 0, 0, 0, \frac{1}{192} (-3 - 5 T) - c_{3,50}, \\
 & \frac{1}{64} (-3 - 5 T + 8 T^2) + \frac{3}{2} (-1 + T) c_{3,50}, \frac{1}{192} (13 - 23 T + 43 T^2 - 33 T^3) - (1 - 2 T + T^2) c_{3,50}, \\
 & \frac{1}{192} (-17 T + 36 T^2 - 39 T^3 + 20 T^4) - \frac{1}{2} (1 - 3 T + 3 T^2 - T^3) c_{3,50}, 0, -\frac{1}{48}, \frac{1}{48} (1 + 9 T), \\
 & \frac{1}{48} (-13 + 13 T - 12 T^2), \frac{1}{16} (5 T - 6 T^2 + 3 T^3) - (-1 + 2 T - T^2) c_{3,50}, 0, 0, -\frac{1}{8}, \frac{1}{32} (6 + 5 T), \\
 & -\frac{7 T}{32} + \frac{3}{2} (-1 + T) c_{3,50}, 0, 0, 0, -\frac{1}{48}, 0, 0, 0, 0, 0, \frac{c_{3,2}}{2} + \frac{c_{3,5}}{2}, -c_{3,2}, \frac{c_{3,2}}{T} + c_{3,5}, 0, -c_{3,5}, 0, \\
 & -\frac{1 - 3 T}{32 T} - \frac{(1 - 3 T) c_{3,10}}{2 T} + \frac{c_{3,11}}{T}, -\frac{-1 + 2 T - T^2}{16 T^2} - \frac{(-1 + 3 T - 2 T^2) c_{3,10}}{2 T^2} - \frac{(1 - T) c_{3,11}}{T^2}, 0, -c_{3,10}, \\
 & -\frac{-1 + T}{16 T} - \frac{(-1 + T) c_{3,10}}{T} - \frac{c_{3,11}}{T}, 0, 0, 0, 0, -\frac{-5 + 17 T}{96 T} + \frac{c_{3,26}}{T}, -\frac{2 - 15 T + 13 T^2}{48 T^2} - \frac{(1 - T) c_{3,26}}{T^2}, \\
 & -\frac{-1 + 8 T - 7 T^2}{48 T^3}, 0, -\frac{1}{16}, -\frac{-7 - 3 T}{32 T}, -\frac{3 - 2 T + 2 T^2}{16 T^2} - \frac{(1 - T) c_{3,26}}{T^2}, 0, 0, -\frac{1}{16}, \\
 & -\frac{1 - T}{8 T} - \frac{c_{3,26}}{T}, 0, 0, 0, 0, 0, -\frac{3 - 11 T}{192 T} + \frac{c_{3,50}}{T}, -\frac{4 + 3 T - 7 T^2}{64 T^2} + \frac{3 (-1 + T) c_{3,50}}{2 T^2}, \\
 & -\frac{-25 + 19 T + T^2 + 5 T^3}{192 T^3} - \frac{(-1 + 2 T - T^2) c_{3,50}}{T^3}, -\frac{16 - 23 T + 12 T^2 - T^3 - 4 T^4}{192 T^4} - \frac{(1 - 3 T + 3 T^2 - T^3) c_{3,50}}{2 T^4}, \\
 & 0, \frac{1}{48}, -\frac{9 + T}{48 T}, -\frac{-12 + 13 T - 13 T^2}{48 T^2}, -\frac{7 - 12 T + 9 T^2 + 2 T^3}{48 T^3} - \frac{(1 - 2 T + T^2) c_{3,50}}{T^3}, 0, \\
 & 0, \frac{1}{8}, -\frac{5 + 6 T}{32 T}, -\frac{-2 - 3 T - 2 T^2}{32 T^2} + \frac{3 (-1 + T) c_{3,50}}{2 T^2}, 0, 0, 0, \frac{1}{48}, -\frac{-1 + T}{24 T} - \frac{c_{3,50}}{T}, \\
 & 0, 0, 0, 0, 0, -\frac{c_{3,2}}{2} - \frac{c_{3,5}}{2}, -c_{3,10}, 0, 0, 0, \frac{c_{3,2}}{2} + \frac{c_{3,5}}{2}, c_{3,10}, 0, 0, 0 \}
 \end{aligned}$$

In[*]:= Cases [{R_{1,2}, R̄_{1,2}, C₁, C̄₁}, (c | d | e | f)_{sk,-}, ∞] // Union

Out[*]:= {c_{3,2}, c_{3,5}, c_{3,10}, c_{3,11}, c_{3,26}, c_{3,50}}

In[*]:= CF [{C₁, (C₁[[3, 4]] /. T → T⁻¹) + C̄₁[[3, 4]], (Rp_{1,2} /. T → T⁻¹) [[3, 4]] + R̄p_{1,2}[[3, 4]] } /.
 {c_{3,26} → -1 / 8, c_{3,50} → 1 / 24, c_{3,11} → 1 / 16 + c_{3,10}}]

Out[*]:= {E_{{1}→{1}}} [√T, 0, ∈Series [0, $\frac{p_1 x_1}{2}$, $-\frac{1}{8} p_1 x_1$, $\frac{1}{2} (-c_{3,2} - c_{3,5}) - p_1 x_1 c_{3,10}$]], 0, 0 }

$$\text{In[*]} := \text{CF}[\text{Rp}_{1,2} / \cdot \{c_{3,26} \rightarrow -1/8, c_{3,50} \rightarrow 1/24, c_{3,11} \rightarrow 1/16 + c_{3,10}\}]$$

$$\begin{aligned} \text{Out[*]} := \mathbb{E}_{\{\} \rightarrow \{1,2\}} \left[1, 0, \text{Series} \left[0, \frac{1}{4} (1-T) p_1^2 x_1 x_2 - \frac{1}{2} p_1 p_2 x_1 x_2 + \frac{1}{4} (-T+T^2) p_1^2 x_2^2 + \frac{1}{2} T p_1 p_2 x_2^2, \right. \right. \\ \frac{1}{16} (1-T) p_1^2 x_1 x_2 + \frac{1}{8} p_1 p_2 x_1 x_2 + \frac{1}{24} (-1+T) p_1^3 x_1^2 x_2 - \frac{1}{8} p_1^2 p_2 x_1^2 x_2 + \\ \frac{1}{16} (2-3T+T^2) p_1^2 x_2^2 - \frac{1}{8} p_1 p_2 x_2^2 + \frac{1}{12} (-2+T+T^2) p_1^3 x_1 x_2^2 + \frac{1}{8} (1+2T) p_1^2 p_2 x_1 x_2^2 - \\ \frac{1}{8} p_1 p_2^2 x_1 x_2^2 + \frac{1}{24} (T+2T^2-3T^3) p_1^3 x_2^3 + \frac{1}{8} (-1+T-2T^2) p_1^2 p_2 x_2^3 + \frac{1}{8} p_1 p_2^2 x_2^3, \\ \frac{1}{96} (17+7T) p_1^3 x_1^2 x_2 + \frac{1}{16} p_1^2 p_2 x_1^2 x_2 + \frac{1}{192} (-11-5T) p_1^4 x_1^3 x_2 - \frac{1}{48} p_1^3 p_2 x_1^3 x_2 + \\ \frac{1}{48} (13-9T-4T^2) p_1^3 x_1 x_2^2 + \frac{1}{32} (-3-7T) p_1^2 p_2 x_1 x_2^2 + \frac{1}{16} p_1 p_2^2 x_1 x_2^2 + \frac{1}{64} (-7-T+8T^2) p_1^4 x_1^2 x_2^2 + \\ \frac{1}{48} (1+9T) p_1^3 p_2 x_1^2 x_2^2 - \frac{1}{8} p_1^2 p_2^2 x_1^2 x_2^2 + \frac{1}{48} (-7T+8T^2-T^3) p_1^3 x_2^3 + \frac{1}{16} (2+T^2) p_1^2 p_2 x_2^3 - \\ \frac{1}{8} p_1 p_2^2 x_2^3 + \frac{1}{192} (5-7T+35T^2-33T^3) p_1^4 x_1 x_2^3 + \frac{1}{48} (-13+13T-12T^2) p_1^3 p_2 x_1 x_2^3 + \\ \frac{1}{32} (6+5T) p_1^2 p_2^2 x_1 x_2^3 - \frac{1}{48} p_1 p_2^3 x_1 x_2^3 + \frac{1}{192} (-4-5T+24T^2-35T^3+20T^4) p_1^4 x_2^4 + \\ \frac{1}{48} (2+11T-16T^2+9T^3) p_1^3 p_2 x_2^4 + \frac{1}{32} (-2-5T) p_1^2 p_2^2 x_2^4 + \frac{1}{24} p_1 p_2^3 x_2^4 + p_1 x_1 c_{3,2} + \\ \left. \frac{1}{2} (-c_{3,2} - c_{3,5}) + p_1 x_2 (-T c_{3,2} - c_{3,5}) + p_2 x_2 c_{3,5} + p_1 p_2 x_1 x_2 c_{3,10} + \frac{1}{16} p_1 p_2 x_2^2 (1 + 16 c_{3,10}) + \right. \\ \left. \frac{1}{32} p_1^2 x_1 x_2 (-3 - T - 48 c_{3,10} - 16 T c_{3,10}) + \frac{1}{16} p_1^2 x_2^2 (-1 + T - 16 c_{3,10} + 8 T c_{3,10} + 8 T^2 c_{3,10}) \right] \end{aligned}$$

```

In[*]:= CF [
  Rp1,2 /. {C3,26 -> -1 / 8, C3,50 -> 1 / 24, C3,11 -> 1 / 16 + C3,10} /. {C3,2 -> 0, C3,5 -> 0, C3,10 -> -1 / 32} ]
Out[*]:= E_{i -> {1,2}} [ 1, 0, Series [ 0, 1/4 (1 - T) p1^2 x1 x2 - 1/2 p1 p2 x1 x2 + 1/4 (-T + T^2) p1^2 x2^2 + 1/2 T p1 p2 x2^2,
  1/16 (1 - T) p1^2 x1 x2 + 1/8 p1 p2 x1 x2 + 1/24 (-1 + T) p1^3 x1^2 x2 - 1/8 p1^2 p2 x1^2 x2 +
  1/16 (2 - 3 T + T^2) p1^2 x2^2 - 1/8 p1 p2 x2^2 + 1/12 (-2 + T + T^2) p1^3 x1 x2^2 + 1/8 (1 + 2 T) p1^2 p2 x1 x2^2 -
  1/8 p1 p2^2 x1 x2^2 + 1/24 (T + 2 T^2 - 3 T^3) p1^3 x2^3 + 1/8 (-1 + T - 2 T^2) p1^2 p2 x2^3 + 1/8 p1 p2^2 x2^3,
  1/64 (-3 - T) p1^2 x1 x2 - 1/32 p1 p2 x1 x2 + 1/96 (17 + 7 T) p1^3 x1^2 x2 + 1/16 p1^2 p2 x1^2 x2 +
  1/192 (-11 - 5 T) p1^4 x1^3 x2 - 1/48 p1^3 p2 x1^3 x2 + 1/64 (-2 + 3 T - T^2) p1^2 x2^2 + 1/32 p1 p2 x2^2 +
  1/48 (13 - 9 T - 4 T^2) p1^3 x1 x2^2 + 1/32 (-3 - 7 T) p1^2 p2 x1 x2^2 + 1/16 p1 p2^2 x1 x2^2 + 1/64 (-7 - T + 8 T^2) p1^4 x1^2 x2^2 +
  1/48 (1 + 9 T) p1^3 p2 x1^2 x2^2 - 1/8 p1^2 p2^2 x1^2 x2^2 + 1/48 (-7 T + 8 T^2 - T^3) p1^3 x2^3 + 1/16 (2 + T^2) p1^2 p2 x2^3 -
  1/8 p1 p2^2 x2^3 + 1/192 (5 - 7 T + 35 T^2 - 33 T^3) p1^4 x1 x2^3 + 1/48 (-13 + 13 T - 12 T^2) p1^3 p2 x1 x2^3 +
  1/32 (6 + 5 T) p1^2 p2^2 x1 x2^3 - 1/48 p1 p2^3 x1 x2^3 + 1/192 (-4 - 5 T + 24 T^2 - 35 T^3 + 20 T^4) p1^4 x2^4 +
  1/48 (2 + 11 T - 16 T^2 + 9 T^3) p1^3 p2 x2^4 + 1/32 (-2 - 5 T) p1^2 p2^2 x2^4 + 1/24 p1 p2^3 x2^4 ] ]

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Slow G

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In[*]:=  $\alpha = 4; \beta = 4;$ 

```

```

In[*]:= rvk = RVK [K]

```

```

Out[*]:= RVK [ {Xm [4, 1], Xm [6, 3], Xm [2, 5]}, {0, 0, 0, -1, 0, 0} ]

```

```

In[*]:= n = Length [rvk [[1]]]

```

```

Out[*]:= 3

```

```

In[*]:=  $\xi = \text{Times} @@ \text{rvk} [[1]] /. \{Xp [i_, j_] \Rightarrow R0_{i,j}, Xm [i_, j_] \Rightarrow \overline{R0}_{i,j}\}$ 

```

```

Out[*]:= E_{i -> {1,2,3,4,5,6}} [ 1/T^{3/2},
  (-1 + 1/T) (-p1 + p4) x1 + (-1 + 1/T) (-p3 + p6) x3 + (-1 + 1/T) (p2 - p5) x5, Series [0, 0] ]

```

In[*]:= $\xi = \mathbb{E}_{\{\} \rightarrow \{\emptyset\} \cup \text{Table}[i-1/2, \{i, 2n+1\}]} [1, \emptyset, \text{eSeries}[\emptyset, p_{\alpha-1/2} x_{\beta-1/2}]]$

Out[*]:= $\mathbb{E}_{\{\} \rightarrow \{\emptyset, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}, 4, \frac{9}{2}, 5, \frac{11}{2}, 6, \frac{13}{2}\}} \left[\frac{1}{T^{3/2}}, \left(-1 + \frac{1}{T}\right) (-p_1 + p_4) x_1 + \left(-1 + \frac{1}{T}\right) (-p_3 + p_6) x_3 + \left(-1 + \frac{1}{T}\right) (p_2 - p_5) x_5, \text{eSeries}[\emptyset, p_{\frac{7}{2}} x_{\frac{7}{2}}] \right]$

In[*]:= **Do**[$\xi = \xi // m_{\emptyset, k \rightarrow \emptyset, \{k, 1/2, 2n+1/2, 1/2\}}$];
 ξ

Out[*]:= $\mathbb{E}_{\{\} \rightarrow \{\emptyset\}} \left[\frac{T^{3/2}}{1 - T + T^2}, \emptyset, \text{eSeries}[\emptyset, \frac{1 - T + T^2 - T^3}{1 - T + T^2} + T p_{\emptyset} x_{\emptyset}] \right]$

In[*]:= **Factor**[$\xi[[3, 2]] /. (x | p)_{\emptyset} \rightarrow \emptyset$]

Out[*]:= $-\frac{(-1 + T)(1 + T^2)}{1 - T + T^2}$

Fast ρ_1

$R_{ij}^S = T^{s/2} (T^s - 1) (p_i - p_j) x_{ij}$
 $x_p = px - 1 \Rightarrow$
 $G_{x_p} = \langle p_x x_p \rangle$ & with effort: $G_{x_{\beta}} = \langle x_{\beta} x_{\beta} \rangle = G_{x_{\beta}} \int dx_{\beta}$
 $G_{x_{\beta}} = 0$
 $X_{ij}^S: \begin{cases} \text{row } i & \tilde{G}_{i,j\beta} - G_{i+1,j\beta} = 0 \Leftrightarrow G_{i\beta} - G_{i+1\beta} = r_{i\beta} \\ \text{row } j & \tilde{G}_{j\beta} - G_{j+1\beta} - (T^s - 1)(G_{i+1,j\beta} - \tilde{G}_{i\beta}) = 0 \end{cases}$
 $\Leftrightarrow T^s G_{j\beta} - G_{j+1\beta} + (1 - T^s) G_{i+1\beta} = T^s r_{j\beta}$
 $B = (\phi | A) \quad G = \begin{pmatrix} 0 & 0 & 0 \\ D & 0 & 0 \end{pmatrix} \quad BG = \begin{pmatrix} I_{2n \times 2n} & 0 \end{pmatrix}$
 $AD = I$

In[*]:= **K = GST48;**
Alexander[K][T]

Out[*]:= $13 - \frac{1}{T^8} + \frac{2}{T^7} - \frac{1}{T^6} - \frac{2}{T^4} + \frac{5}{T^3} - \frac{2}{T^2} - \frac{7}{T} - 7T - 2T^2 + 5T^3 - 2T^4 - T^6 + 2T^7 - T^8$

In[*]:= CF /@ {Rp_{1,2}, R̄p_{1,2}, C₁, C̄₁}

$$\text{Out[*]} = \left\{ \mathbb{E}_{\{\} \rightarrow \{1,2\}} \left[1, \theta, \in \text{Series} \left[\theta, \frac{1}{4} (1 - T) p_1^2 x_1 x_2 - \frac{1}{2} p_1 p_2 x_1 x_2 + \frac{1}{4} (-T + T^2) p_1^2 x_2^2 + \frac{1}{2} T p_1 p_2 x_2^2 \right] \right], \right. \\ \mathbb{E}_{\{\} \rightarrow \{1,2\}} \left[1, \theta, \in \text{Series} \left[\theta, \frac{(1 - T) p_1^2 x_1 x_2}{4 T} + \frac{1}{2} p_1 p_2 x_1 x_2 + \frac{(-1 + T) p_1^2 x_2^2}{4 T^2} - \frac{p_1 p_2 x_2^2}{2 T} \right] \right], \\ \left. \mathbb{E}_{\{\} \rightarrow \{1\}} \left[\sqrt{T}, \theta, \in \text{Series} \left[\theta, \frac{p_1 x_1}{2} \right] \right], \mathbb{E}_{\{\} \rightarrow \{1\}} \left[\frac{1}{\sqrt{T}}, \theta, \in \text{Series} \left[\theta, -\frac{1}{2} p_1 x_1 \right] \right] \right\}$$

```
PAB1[K_] := Module[{rvk, n, B, c, s, i, j, rho0, G, rho1},
  rvk = RVK[K]; n = Length[rvk[[1]]; B = Table[0, {2 n, 2 n + 1};
  Do[
    s = If[Head[c] === Xp, 1, -1]; {i, j} = List@@c;
    B[[i, {i, i + 1}]] = {1, -1}; B[[j, {j, j + 1, i + 1}]] = {1, -Ts, T-s - 1},
    {c, rvk[[1]]};
  rho0 = Det[B[;;, 2 ;;]] // Factor;
  G = Table[0, {2 n, 2 n}; G[;;, ;, 2 n - 1] = Factor@Inverse[B[;;, 2 ;;]][[;;, 2 ;; 2 n]];
  rho1 = Factor@Plus[
    Sum[
      s = If[Head[c] === Xp, 1, -1]; {i, j} = List@@c;
      s \left( \frac{1 - T^s}{2} G[[i, i]] G[[i, j]] - \frac{1}{2} (G[[i, i]] G[[j, j]] + G[[i, j]] G[[j, i]]) + \right. \\ \left. \frac{T^{2s} - T^s}{2} G[[i, j]]^2 + T^s (G[[i, j]] G[[j, j]]) \right), {c, rvk[[1]]},
    Sum[\frac{rvk[[2, k]]}{2} G[[k - 1, k - 1]], {k, 2, 2 n} ] ];
  {rho0, rho1} ]
```

In[*]:= Timing@PAB1[K]

$$\text{Out[*]} = \left\{ 78.6094, \left\{ -\frac{(-1 + 2 T - T^2 - T^3 + 2 T^4 - T^5 + T^8) (-1 + T^3 - 2 T^4 + T^5 + T^6 - 2 T^7 + T^8)}{T^7}, \right. \right. \\ \left. - \left(\left((-1 + T) (13 - 45 T + 71 T^2 - 71 T^3 + 58 T^4 - 35 T^5 - 39 T^6 + 101 T^7 + 37 T^8 - 335 T^9 + 372 T^{10} + 56 T^{11} - \right. \right. \right. \\ \left. \left. \left. 506 T^{12} + 478 T^{13} - 114 T^{14} - 81 T^{15} - 31 T^{16} + 204 T^{17} - 270 T^{18} + 210 T^{19} + 8 T^{20} - 328 T^{21} + \right. \right. \right. \\ \left. \left. \left. 481 T^{22} - 311 T^{23} - 7 T^{24} + 169 T^{25} - 115 T^{26} - 10 T^{27} + 59 T^{28} - 31 T^{29} + T^{30} + 3 T^{31} \right) \right) \right) / \\ \left. \left(2 (-1 + 2 T - T^2 - T^3 + 2 T^4 - T^5 + T^8)^2 (-1 + T^3 - 2 T^4 + T^5 + T^6 - 2 T^7 + T^8)^2 \right) \right\} \right\}$$

In[*]:= **Timing@Factor@ZF[K][[3, 2]]**

Out[*]:= {3489.64,

$$- \left(\left((-1 + T) \left(13 - 45 T + 71 T^2 - 71 T^3 + 58 T^4 - 35 T^5 - 39 T^6 + 101 T^7 + 37 T^8 - 335 T^9 + 372 T^{10} + 56 T^{11} - 506 T^{12} + 478 T^{13} - 114 T^{14} - 81 T^{15} - 31 T^{16} + 204 T^{17} - 270 T^{18} + 210 T^{19} + 8 T^{20} - 328 T^{21} + 481 T^{22} - 311 T^{23} - 7 T^{24} + 169 T^{25} - 115 T^{26} - 10 T^{27} + 59 T^{28} - 31 T^{29} + T^{30} + 3 T^{31} \right) \right) / \left(2 \left(-1 + 2 T - T^2 - T^3 + 2 T^4 - T^5 + T^8 \right)^2 \left(-1 + T^3 - 2 T^4 + T^5 + T^6 - 2 T^7 + T^8 \right)^2 \right) \right) \}$$