Exponential Zipping

March 2, 2019 6:31 PM

Let [ ] stand for "partial zipping":

$$[f(\zeta,z)]_\lambda = \langle f(\zeta, z+z')\rangle_\lambda /_{\cdot}\, z' \to z$$

$$\langle f(\zeta,z)\rangle_\lambda = [f(\zeta,z)]_\lambda /_{\cdot}\, z \to 0$$

$[\ ]_\lambda$ and $\langle\ \rangle_\lambda$ means "coupling strength is λ".

Proposition. Suppose (the 1-variable case)

$$\exp(P_\lambda) = [\exp(f(\zeta,z))]_\lambda$$

Then $P_0 = f$

and $\partial_\lambda P_\lambda = (\partial_\zeta \partial_z P_\lambda) + (\partial_\zeta P_\lambda)(\partial_z P_\lambda)$

Proof $\qquad [f]_\lambda = e^{\lambda \partial_\zeta \partial_z} f$

$$\partial_\lambda P_\lambda = \partial_\lambda \log [e^f]_\lambda = \partial_\lambda e^{\lambda\partial_\zeta\partial_z} e^f \big/ e^{P_\lambda}$$

$$= e^{-P_\lambda} \partial_\zeta \partial_z \left(e^{\lambda\partial_\zeta\partial_z} e^f\right) = e^{-P_\lambda}\partial_\zeta\partial_z e^{P_\lambda}$$

$$= e^{-P_\lambda}\partial_\zeta\left((\partial_z P_\lambda)\cdot e^{P_\lambda}\right)$$

$$= e^{-P_\lambda}\left(e^{P_\lambda}(\partial_\zeta\partial_z P_\lambda) + (\partial_z P_\lambda)(\partial_\zeta P_\lambda)\right)$$

$$= (\partial_\zeta \partial_z P_\lambda) + (\partial_z P_\lambda)(\partial_\zeta P_\lambda) \qquad \square$$