Exponential Zipping

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Let $\square$ stand for “partial zipping”. \[ f \] means
\[ \langle f(\lambda, z) \rangle_\lambda = \langle f(\lambda, z + z')_\lambda / z' \rightarrow z \]
\[ \langle f(\lambda, z) \rangle_\lambda = \langle f(\lambda, z) \rangle_\lambda / z \rightarrow 0 \]

Proposition. Suppose (the 1-variable case)
\[ \exp(p_\lambda) = \langle \exp(f(\lambda, z)) \rangle_\lambda \]

Then $p_0 = f$

and $\exists x p_\lambda = (\exists x z p_\lambda) + (2 x p_\lambda)(\exists z p_\lambda)$

**Proof**

\[ \langle f \rangle_\lambda = e^x \partial_x \langle f \rangle_\lambda \]

\[ \partial_x p_\lambda = \partial_x \log \langle e^x \rangle_\lambda = \partial_x e^x \partial_x \langle f \rangle_\lambda / \partial x \]

\[ = e^{-p_\lambda} \partial_x (e^{x \partial_x} \langle f \rangle_\lambda) = e^{-p_\lambda} \partial_x \partial_x \langle f \rangle_\lambda \]

\[ = e^{-p_\lambda} \partial_x (e^{x \partial_x} \langle f \rangle_\lambda) = e^{-p_\lambda} (e^{x \partial_x} (\partial_x p_\lambda) + (\partial_x p_\lambda) (\partial_x p_\lambda)) \]

\[ = (\partial_x \partial_x p_\lambda) + (\partial_x p_\lambda)(\partial_x p_\lambda) \quad \Box \]