EDDO at k=0 (multi-variable, added 2019-06)

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Compute $\left[e^{F(z)\zeta_j}\right]_\lambda := e^{\lambda \partial_{z^i}\partial_{\zeta_i}} e^{F^j(z)\zeta_j} =: e^{W_\lambda + F^j_\lambda \zeta_j}$

W, F^j depend on z, λ but not on ζ

$$(\partial_\lambda W + \zeta_j \partial_\lambda F^j) e^{W_\lambda + F^j_\lambda \zeta_j} = \partial_\lambda e^{W + F^j\zeta_j} =$$
$$= \partial_\lambda e^{\lambda\partial_{z^i}\partial_{\zeta_i}} e^{F^j(z)\zeta_j} = \partial_{z^i}\partial_{\zeta_i} e^{\lambda\partial_{z^i}\partial_{\zeta_i}} e^{F^j(z)\zeta_j}$$
$$= \partial_{z^i}\partial_{\zeta_i} e^{W+F^j\zeta_j}$$
$$= \partial_{z^i}\left(F^i e^{W+F^j\zeta_j}\right)$$
$$= \left(\partial_{z^i}F^i + F^i\partial_{z^i}W + F^i\zeta_j\partial_{z^i}F^j\right) e^{W+F^j_\lambda\zeta_j}$$

$$\boxed{F^j(\lambda=0) = F^j}$$
initial condition

So $\partial_\lambda W + \zeta_j \partial_\lambda F^j = \partial_{z^i}F^i + F^i\partial_{z^i}W + F^i\zeta_j\partial_{z^i}F^j$

So $\boxed{\partial_\lambda F^j = F^i \partial_{z_i} F^j}$ $*$

one variable from here on

and $\partial_\lambda W = \partial_z F + F\partial_z W$

characteristics for $*$:

$$\frac{d\lambda}{dt} = 1 \qquad \frac{dz}{dt} = -\phi \qquad \frac{d\phi}{dt} = 0$$

$$\lambda = t + a \qquad z = -\phi t + b \qquad \phi = c$$

$F(\lambda, z) =$
$F(0, \phi\lambda + z)$

take $a=\lambda$, $b=z$
$t = -\lambda$

If $F = f(\lambda)z$, get
$f'z = fzf$
$f' = f^2$
$f = \frac{1}{1-\lambda} \qquad f' = \frac{1}{(1-\lambda)^2}$

$\partial_\lambda W = \frac{1}{1-\lambda} + \frac{z}{1-\lambda}\partial_z W$

$W = \log(1-\lambda)^{-1}$