A Verma Module

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$t = \epsilon a - \gamma b$

At $\hbar = 0$, $\mathcal{U}_{0;\gamma\epsilon} = \langle t, y, a, x\rangle / ([t,\cdot] = 0,\ [a,x] = \gamma x,\ [a,y] = -\gamma y,\ [x,y] = 2\epsilon a - t$ with $\Delta(t,y,a,x) = (t_1+t_2, y_1+y_2, a_1+a_2, x_1+x_2)$ and $\theta(y,b,a,x) = (-x,-b,-a,-y)$.

In $\mathcal{U}/\mathcal{U}\langle y, a-\lambda\rangle =: V$:

$V = \langle x^n\rangle$

x acts by $\hat{x}$

a acts by $\lambda + \gamma\hat{x}\partial_x$

y acts by

$(t - 2\epsilon\lambda)\partial_x - \epsilon\gamma\hat{x}\partial_x^2$

$ax = x(a+\gamma)$

$ax^n = x^n(a+n\gamma) = (\lambda+n\gamma)x^n$

$yx = xy + t - 2\epsilon a$

$yx^n = x^n y + nt x^{n-1} - 2\epsilon n x^{n-1} a - 2\epsilon\frac{n(n-1)}{2}\gamma x^{n-1}$

$x^2y^2 \to (\qquad)\hat{x}^2\,{}^2$

$ax = x(a+\gamma) \Longrightarrow ax^n = x^n(a+n\gamma)$

$\Downarrow$

$xa = (a-\gamma)x \Longrightarrow xa^n = (a-\gamma)^n x$