Convergence

Convergence of the swap for $e^{\hbar xy}$

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(170625) $\mathcal{U}_{\hbar;\gamma\epsilon}$ conventions: $q = e^{\hbar\gamma\epsilon}$, $H = \langle a, x\rangle / ([a, x] = \gamma x)$ with

$$A = e^{-\hbar\epsilon a}, \quad xA = qAx, \quad S(a, A, x) = (-a, A^{-1}, -A^{-1}x),$$
$$\Delta(a, A, x) = (a_1 + a_2, A_1A_2, x_1 + A_1x_2)$$

and dual $H^* = \langle b, y\rangle / ([b, y] = -\epsilon y)$ with

$$B = e^{-\hbar\gamma b}, \quad By = qyB, \quad S(b, B, y) = (-b, B^{-1}, -yB^{-1}),$$
$$\Delta(b, B, y) = (b_1 + b_2, B_1B_2, y_1B_2 + y_2).$$

Pairing by $(a, x)^* = \hbar(b, y)$ ($\Rightarrow \langle B, A\rangle = q$) making $\langle y^l b^i, a^j x^k\rangle = \delta_{ij}\delta_{kl}\hbar^{-(j+k)} j![k]_q!$ so $R = \sum \frac{\hbar^{j+k} y^k b^j \otimes a^j x^k}{j![k]_q!}$. Then $\mathcal{U} = H^{*cop} \otimes H$ with $(\phi f)(\psi g) = \langle \psi_1 S^{-1} f_3\rangle\langle\psi_3, f_1\rangle(\phi\psi_2)(f_2 g)$. With the central $t := \epsilon a - \gamma b$, $T := e^{\hbar t} = A^{-1}B$ get

$$[a, y] = -\gamma y, \quad [b, x] = \epsilon x, \quad xy - qyx = (1 - \underbrace{TA^2}_{BA})/\hbar.$$

Cartan: $\theta(y, b, a, x) = (-B^{-1}T^{1/2}x, -b, -a, -A^{-1}T^{-1/2}y)$.
At $\epsilon = 0$, $\mathcal{U}_{\hbar;\gamma 0} = \langle b, y, a, x\rangle / ([b, \cdot] = 0, [a, x] = \gamma x, [a, y] = -\gamma y, [x, y] = (1 - e^{-\hbar\gamma b})/\hbar)$ with $\Delta(b, y, a, x) = (b_1 + b_2, y_1 + e^{-\hbar\gamma b_1} y_2, a_1 + a_2, x_1 + x_2)$ and $\theta(y, b, a, x) = (-e^{\hbar\gamma b/2}x, -b, -a, -e^{\hbar\gamma b/2}y)$.

$$x \mapsto x\cdot 1\cdot 1 + A\cdot x\cdot 1 + A\cdot A\cdot x$$
$$y \mapsto y\cdot B\cdot B + 1\cdot y\cdot B + 1\cdot 1\cdot y$$

So $\underset{F\ \ \psi}{sw(x, y)} \sim \hbar^{-1}(-1 + BA) + qyx$

The fact that there are no $\hbar^{-1}$ in the end result remains a miracle.