# Simplifying yaxyax

October 16, 2017 2:41 PM

**(171012)** Talks/LesDiablerets-1708, esp. PBWDemo.nb, verifications 2017-10/Phi2CR-Classical.nb: In $\hat{\mathcal{U}}(\mathfrak{g}^\epsilon) = \langle t,y,a,x\rangle/([t,\cdot]=0,\ [a,x]=x,\ [a,y]=-y,\ [x,y]=t-2\epsilon a)$, we have $\prod_{i=1}^2 \mathbb{e}^{\tau_i t}\mathbb{e}^{\eta_i y}\mathbb{e}^{\alpha_i a}\mathbb{e}^{\xi_i x} = \mathbb{e}^{\tau t}\mathbb{e}^{\eta y}\mathbb{e}^{\alpha a}\mathbb{e}^{\xi x}$, with

$$\tau = \tau_1+\tau_2-\frac{\log(1-\epsilon\eta_2\xi_1)}{\epsilon} = \tau_1+\tau_2+\eta_2\xi_1+\frac{\epsilon}{2}\eta_2^2\xi_1^2+\ldots,$$

$$\eta = \eta_1+\frac{\mathbb{e}^{-\alpha_1}\eta_2}{(1-\epsilon\eta_2\xi_1)} = \eta_1+\mathbb{e}^{-\alpha_1}\eta_2+\epsilon\mathbb{e}^{-\alpha_1}\eta_2^2\xi_1+\ldots,$$

$$\alpha = \alpha_1+\alpha_2+2\log(1-\epsilon\eta_2\xi_1) = \alpha_1+\alpha_2-2\epsilon\eta_2\xi_1+\ldots,$$

$$\xi = \frac{\mathbb{e}^{-\alpha_2}\xi_1}{(1-\epsilon\eta_2\xi_1)}+\xi_2 = \mathbb{e}^{-\alpha_2}\xi_1+\xi_2+\epsilon\mathbb{e}^{-\alpha_2}\eta_2\xi_1^2+\ldots.$$

With $T=e^{\tau}$, $A=e^{\alpha}$, get

$$T=T_1T_2(1-\epsilon\eta_2\xi_1)^{-1/\epsilon}$$

$$\eta=\eta_1+\frac{A_1^{-1}\eta_2}{(1-\epsilon\eta_2\xi_1}$$

$$A=A_1A_2+(1-\epsilon\eta_2\xi_1)^2$$

$$\xi=\frac{A_2^{-1}\xi_1}{(1-\epsilon\eta_2\xi_1)}+\xi_2$$

$$\prod_{i=1}^2 T_i^t e^{\eta_i y}A_i^a e^{\xi_i x} = T^t e^{\eta y}A^a e^{\xi x}$$

In a rep., $T^t \sim \begin{pmatrix} T & \\ & T \end{pmatrix}$ $A^a \sim \begin{pmatrix} A & \\ & A^{-1} \end{pmatrix}$