Scratch

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$$f(t+1)-f(t)-\partial_x^2 f = g(t)$$

$$e^{(t-2a\epsilon)\hbar}+\hbar\left(e^z\,g[z,-1+a]-\left(e^{z+\epsilon\hbar}+t-2a\epsilon\right)g[z,a]+(-t+\epsilon+2a\epsilon)\,g^{(1,0)}[z,a]+\epsilon\,g^{(2,0)}[z,a]\right)==1$$

$$f(t+1)-f(t)=e^{\epsilon t} \qquad e^{\epsilon(t+1)}-e^{\epsilon t}=$$

$$f=\frac{1}{e^{\epsilon}-1}e^{\epsilon t} \qquad e^{\epsilon t}(e^{\epsilon}-1)$$

$$\binom{a}{k}:=\frac{a(a-1)\dots(a-k+1)}{k!} \quad \text{satisfies} \quad \binom{a}{k}-\binom{a-1}{k}=\binom{a-1}{k-1}$$

$$a_k := a(a-1)\dots(a-k+1) \quad \text{satisfies} \quad \delta_a a_k = k a_{k-1}$$

<u>Q.</u> What's the kernel of

$$\delta_x f+\partial_y f = f(x+1,y)-f(x,y)+\partial_y f(x,y) \quad ?$$

$$(\partial_x+\partial_y)(x-y)^n=\sum_{k=0}^{n}(-1)^k\binom{n}{k}x^k y^{n-k}\left(\frac{k}{x}+\frac{n-k}{y}\right)$$

$$\sum_{k=0}^{n}\binom{n}{k}x_k y^{n-k} \,//\, \delta_x-\partial_y = \sum_{k=0}^{n}\binom{n}{k}\left[k x_{k-1}y^{n-k}-(n-k)x_k y^{n-k-1}\right]$$

$$\sum_{k=0}^{n}\binom{n}{k}x_k y^{n-k}=$$

$$\sum_{j=0}^{k}\binom{x}{j}\frac{y^j}{(n-j)!}=\sum_{j=0}^{k}\frac{x(x-1)\dots(x-j+1)\,y^j}{j!\,(n-j)!}$$

$$\sum_{j=0} \binom{x}{j} \frac{y^j}{(n-j)!} = \sum_{j=0} \frac{x(x-1)\dots(x-j+1)\,y^j}{j!\,(n-j)!}$$

<u>Aside</u> $x_k = \lambda^{k-x}(\partial_\lambda)^k \lambda^x$