A decreasing filtration on U(solvable)

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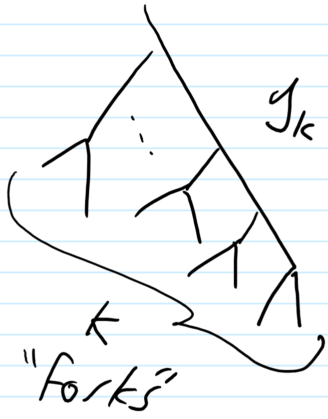
(170224) $\mathfrak{g}$ solvable $\Leftrightarrow$ $[\mathfrak{g},\mathfrak{g}]$ nilpotent $\overset{?}{\Leftrightarrow}$ $\mathfrak{g} \cong \mathfrak{a} \ltimes \mathfrak{n}$ with Abelian $\mathfrak{a}$ and nilpotent $\mathfrak{n}$. Also, $\mathfrak{g}$ solvable $\Leftrightarrow$ there is a finite decreasing filtration $(\mathfrak{g}_k)_{k\geqslant 0}$, $\mathfrak{g}_0 = \mathfrak{g}$, with $\mathfrak{g}_0/\mathfrak{g}_1$ Abelian and $[\mathfrak{g}_k, \mathfrak{g}_l] \subset \mathfrak{g}_{k+l}$. Then $\mathcal{U}(\mathfrak{g})$ also has a multiplicative decreasing filtration.



$\mathfrak{g}_0 = \mathfrak{g}$; $\mathfrak{g}_1 = [\mathfrak{g},\mathfrak{g}]$; $\mathfrak{g}_{k>1} = [\mathfrak{g}, \mathfrak{g}_{k-1}]$

$U(\mathfrak{g})_k$ = what can be written using k forks

$= \langle [\mathfrak{g},\mathfrak{g}]^{\otimes k} \rangle_{U(\mathfrak{g})}$

Claim (?) If $\mathfrak{g}$ is solvable, then there is a descending filtration U_d on $U = U(\mathfrak{g})^{\otimes S}$ so that 1. U/U_d is f.d.

2. $m^{ij}_k(U_d) \subset U_{l(d)}$, with $l(d) \uparrow \infty$ as $d \uparrow \infty$.

3. [Some property about "the pairing of very high things with very low things always vanishes"]